Basic Terminology

- A **treatment** (factor) is a variable that the experimenter defines and controls so that its effect can be evaluated (a generic term).
- **Treatment levels**: the different values of a treatment.
- **Response variable (Y)**: the variable that is measured.
- **Experimental unit**: the unit to which one application of the treatment is applied. (Each experimental unit receives only one treatment level.)
- **Sampling unit**: the unit on which the effect of the treatment is measured. (each sampling unit provides one measurement of the response variable.)

Example 1: one-way ANOVA

- A field experiment is conducted to compare the yield of three varieties of corn (A, B, C). A field containing 30 plots of land is used for the experiments. Each variety is planted on 10 randomly selected plots. The yield for each plot is measured at the time of harvest.

- **Treatment**: corn variety
- **Treatment levels**: the three corn varieties A, B, C
- **Response variable**: yield
- **Experimental unit**: plot
- **Sampling unit**: plot

Factorial design

- two or more treatments **cross classified**

- An experiment is conducted to determine the optimal time and temperature combination for baking a cake. The response variable is taste. Consider 3 baking temperatures (275ºF, 300ºF, 325ºF) and 2 baking times (25 and 30 minutes). Four batches of cake will be baked separately at each combination of baking time and temperature.

- Treatments: time and temperature
- Treatment levels for temperature: 275ºF, 300ºF, 325ºF
- Treatment levels for time: 25 and 30 minutes
- Response variable: taste
- Experimental Unit: cake
- Sampling unit: cake

Example 3

- An experiment is conducted to gauge the effect of soil temperature on the number of seeds produced by soybean plants. Six plants are grown in each of three baths. Three temperatures (low, medium, and high) are assigned at random to the three baths. At the end of the growing season the number of seeds produced by each of the 18 plants is recorded.

- Treatment: soil temperature
- Treatment levels: low, medium, and high temperatures
- Response variable: number of seeds
- Experimental unit: bath
- Sampling unit: plant

Randomized block design

- The experimental units are first sorted into homogeneous groups (blocks), and the treatments are then assigned at random within the blocks.

- Example: In an experiment on the effects of four levels of newspaper advertising saturation on sales volume, the experimental unit is a city, and 16 cities are available for the study. Size of city usually is highly correlated with the response variable, sales volume. Hence, it is desirable to block the 16 cities into four groups of four cities each, according to population size.

- Different fields/greenhouses/incubators

Fixed effects

1. All levels of the treatment are included.
   - Male and female
   - Before and after
   - Large, medium, and small

2. All levels of interest are included.
   - Compare the effects of three specific dosages of a drug (100mg, 200mg, 300mg)
   - Compare the sodium content of the top 6 best selling beers of 2015

$$H_0\colon \mu_1 = \cdots = \mu_6$$

Usually treatments are fixed!

Random effects

- Treatment has many possible levels, **interest is in all possible levels**. But only a random sample of levels is included in the experiment.
- Example: all brands of American beer have the same sodium content? For 6 randomly selected brands of beer, the sodium content of 10 12 ounce bottles was recorded.

$$H_0\colon \sigma_\tau{}^2 = 0$$

We are not interested in any difference between the 6 brands used in the experiment. If we were interested in the 6 brands, we would use a fixed effects model.

ANOVA Source Table

- EMS: expected mean squares

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	4	838.5976	209.6494	15.38	0.0001
Error	20	272.6680	13.6334		
Corrected Total	24	1111.2656			

$\sigma^2 + n\frac{\sum \tau_i^2}{t-1}$ if fixed

$\sigma^2 + n\sigma_\tau^2$ if random

σ^2

Latin Square Design (LSD)

- Two sources of blocking (rows and columns)
- Each treatment occurs once and only once in each row and column
- Examples of a 4x4 and a 6x6 Latin square designs:

		Column			
		1	2	3	4
Rows	1	A	C	B	D
	2	D	A	C	B
	3	B	D	A	C
	4	C	B	D	A

		Column					
		1	2	3	4	5	6
Rows	1	A	B	C	D	E	F
	2	B	C	D	E	F	A
	3	C	D	E	F	A	B
	4	D	E	F	A	B	C
	5	E	F	A	B	C	D
	6	F	A	B	C	D	E

Example I of LSD

Four machines (A, B, C, D) are to be tested to see whether they differ significantly in their ability to produce a manufactured part. Different operators and different time periods in the work day are known to have an effect on production.

		Operator			
		1	2	3	4
	1	C	D	A	B
Time	2	D	A	B	C
Period	3	B	C	D	A
	4	A	B	C	D

Example II of LSD

Four cars and **four drivers** are employed in a study of **four gasoline additives**(*A*, *B*, *C*, *D*). It would be desirable to eliminate both the car-to-car and driver-to-driver variations when comparing the additives.

	cars			
drivers	1	2	3	4
1	A=24	B=26	D=20	C=25
2	D=23	C=26	A=20	B=27
3	B=15	D=13	C=16	A=16
4	C=17	A=15	B=20	D=20

Split Plot Design

- A design with at least 2 treatments, where experimental units wrt treatments differ in size.
- Assign the levels of one treatment to **main plots** arranged in a CRD or RBD, and then assign the levels of a second treatment to **subplots** within each main plot.
- Main plot: larger experimental unit
- Subplot: experimental units that the whole plot is split into

Split Plot Design

Randomization is two-stage

1. Levels of treatment A are randomized over the main plots

2. Levels of treatment B are randomized over the subplots within each main plot.

Examples of Split Plot Design

1. Irrigation levels are applied to large areas, and factors like fertilizers are assigned to smaller areas within particular irrigation treatments.

2. An experiment is performed to relate growth of plants to different temperature regimes and amounts of a plant food. Temperature regimes are provided by specially constructed environmental chambers, which accommodate large numbers of plant beds, while different plant foods can be used on individual plant beds.

Split plot designs occur when different factors in an experiment require different sizes of experimental units.

Repeated Measures Design

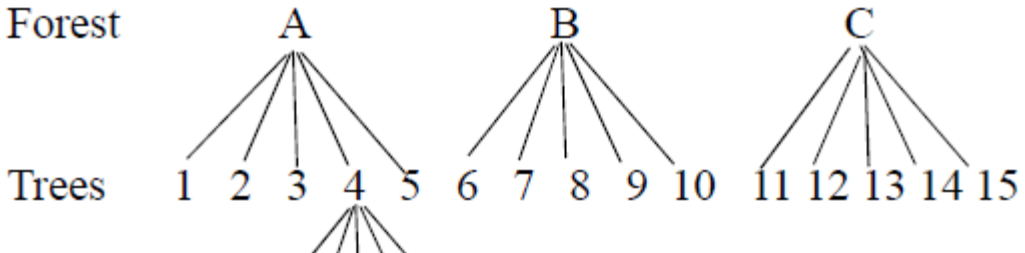
Multiple measurements are made on the same experimental units over a period of time.

Example 1: in evaluating treatments for the relief of asthma FEV1 may be measured before and after intervention.

Example 2: subjects in the study participated in one of three different weightlifting programs, and their strength was measured once every other day for two weeks after they began the program.

Nested Designs

- A forest geneticist collects 5 seeds from 5 trees in each of 3 forests. The seeds are germinated in a greenhouse and the seedlings are measured for height growth.



Total of 75 seedlings.