

Expected mean squares (EMS) for a two-way Analysis of Variance or Completely Randomized Design (CRD) with a factorial treatment arrangement when both treatment effects are fixed.

| | |
|-------------------------------------|--|
| Fixed effect Model Factorial | $Y_{ijk} = \mu + \tau_{1i} + \tau_{2j} + \tau_{1\tau_{2ij}} + \varepsilon_{ijk}$ |
| Source | Expected Mean Squares |
| Treatment1 | $\sigma^2 + Q_{\tau_1}$ |
| Treatment2 | $\sigma^2 + Q_{\tau_2}$ |
| Treatment1 * Treatment2 interaction | $\sigma^2 + Q_{\tau_1\tau_2}$ |
| Error (replicates within) | σ^2 |

Expected mean squares for a two-way Analysis of Variance or Completely Randomized Design (CRD) with a factorial treatment arrangement when one treatment effect is fixed and one is random.

| | |
|-------------------------------------|---|
| Mixed Model Factorial | $Y_{ijk} = \mu + \tau_i + \beta_j + \tau\beta_{ij} + \varepsilon_{ijk}$ |
| Source | Expected Mean Squares |
| Treatment1 | $\sigma^2 + nd\sigma_{\tau_1\tau_2}^2 + Q_{\tau_1}$ |
| Treatment2 | $\sigma^2 + nd\sigma_{\tau_1\tau_2}^2 + nd\sigma_{\tau_2}^2$ |
| Treatment1 * Treatment2 interaction | $\sigma^2 + nd\sigma_{\tau_1\tau_2}^2$ |
| Error (replicates within) | σ^2 |

This is not a very common problem, since many analyses have only fixed effects; random treatment effects are not so common. However, the problem is prevalent in Randomized Block Designs (RBD) since the blocks are random.

Expected mean squares for a one-way Completely Randomized Block Design (RBD) with a single factor treatment arrangement (either fixed or random).

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|---------------------------------------|--|
| Randomized Block Design with Reps | $Y_{ijk} = \mu + \tau_i + \beta_j + \tau\beta_{ij} + \varepsilon_{ijl}$ |
| Source | Expected Mean Squares |
| Treatment | $\sigma^2 + n\sigma_{\tau\beta}^2 + nb\sigma_{\tau}^2$ (random) $\sigma^2 + n\sigma_{\tau\beta}^2 + nb(\sum\tau_i^2/(t-1))$ (fixed) |
| Block | $\sigma^2 + n\sigma_{\tau\beta}^2 + nt\sigma_{\beta}^2$ |
| Treatment*Block (experimental error) | $\sigma^2 + n\sigma_{\tau\beta}^2$ |
| Rep(Treatment*Block) = sampling error | σ^2 |

How is MIXED better?

- 1) GLM and MIXED can both do regression type analyses, though fewer diagnostics are available. PROC REG cannot easily do Analysis of Variance.
- 2) PROC MIXED does not represent a better version of doing ANOVA; it is a whole new way of doing ANOVA. You must think differently.
- 3) Why is PROC MIXED better than PROC GLM?
 - a) PROC GLM is correct only if σ_{ϵ}^2 is the only random effect.
 - b) MIXED will readily handle unbalanced data and give the correct tests, PROC GLM will not in some cases.
 - c) MIXED can test and fit non-homogeneous variance, PROC GLM will not.
 - d) MIXED correctly calculates subplot standard errors, PROC GLM will not.
 - e) MIXED will correctly determine the error term for testing each term in the model, and correctly test LSMeans (even with non-homogeneous variance), PROC GLM will not.
 - f) MIXED will calculate predicted means of fixed effects (zeroing out the random effects). This is called broad inference space, PROC GLM will not.
 - g) MIXED can fit a wide range of covariance structures, and has special provisions for repeated measures designs. Gone are the Huynh - Feldt and Greenhouse - Geisser adjustments of the archaic GLM procedure.
- 4) How is PROC MIXED different from the ancient PROC GLM?
 - a) MIXED is calculated differently (iterative solution, maximum likelihood if normal) versus the least squares of GLM.
 - b) PROC MIXED is structured differently
 - (1) no output statement, but does have OUTP and OUTPM.
 - (2) MIXED has FIXED effects only in the model and RANDOM effects in a separate random statement (GLM has all effects in the model and a primitive version of the random statement).
 - (3) MIXED has a REPEATED statement that can fit various covariance structures.
 - (4) MIXED has no MEANS statement, use LSMeans instead. Use a macro by A. Saxton to get sorted means with letters designating significant differences.

5) Other advantages.

The MIXED approach to ANOVA is adaptable, and can be used to fit nonlinear models (PROC NLMixed) and generalized mixed models (PROC GLIMMIX).

SPSS does mixed model analysis (see MIXED MODELS). However, the offering is not as comprehensive as SAS. For example it does not approximate d.f. for non-homogeneous tests (e.g. DDFM = KenwardRoger or Satterthwaite).

The EMMMeans in SPSS do not alert the user to missing cells (nonestimable functions) as does SAS LSMeans. It averages the available means.

6) We have seen the assumption of “homogeneous variance” relaxed in PROC MIXED, as this procedure will fit and test treatments with different variances. Some aspects of the assumption of independence cannot be relaxed, but a wide selection of covariance structures allows the user to address some aspects of this assumption as well. What if you don’t have normally distributed data? Soon to be available, SAS PROC GLIMMIX. This procedure will fit various distributions as residuals.

| DIST= | Distribution | Function |
|-----------------------------------|-------------------------------|-------------------------------------|
| BINARY | binary | logit |
| BINOMIAL BIN B | binomial | logit |
| EXPONENTIAL EXPO | exponential | log |
| GAMMA GAM | gamma | log |
| GAUSSIAN G NORMAL N | normal | identity |
| GEOMETRIC GEOM | geometric | log |
| INVGAUSS IGAUSSIAN IG | inverse Gaussian | inverse squared (power(-2)) |
| LOGNORMAL LOGN | log-normal | identity |
| MULTINOMIAL MULTI MULT | multinomial cumulative | logit |
| NEGBINOMIAL NEGBIN NB | negative binomial | log |
| POISSON POI P | Poisson | log |
| TCENTRAL TDIST T | t | identity |

Both GLM and MIXED share the following statements as part of their syntax, and functioning is similar for both: CLASS, BY, ID, and WEIGHT.

PROC GLM syntax (partial listing)

```
PROC GLM < options > ;  
  MODEL dependents=independents < / options > ;  
  ABSORB variables ;  
  FREQ variable ;  
  CONTRAST 'label' effect values < ... effect values > < / options > ;  
  ESTIMATE 'label' effect values < ... effect values > < / options > ;  
  LSMEANS effects < / options > ;  
  MANOVA < test-options >< / detail-options > ;  
  MEANS effects < / options > ;  
  OUTPUT < OUT=SAS-data-set >  
    keyword=names < ... keyword=names > < / option > ;  
  RANDOM effects < / options > ;  
  REPEATED factor-specification < / options > ;  
  TEST < H=effects > E=effect < / options > ;
```

PROC MIXED syntax (partial listing)

```
PROC MIXED < options > ;  
  CLASS variables ;  
  MODEL dependent = < fixed-effects > < / options > ;  
  RANDOM random-effects < / options > ;  
  REPEATED < repeated-effect > < / options > ;  
  PARMS (value-list) ... < / options > ;  
  PRIOR < distribution > < / options > ;  
  CONTRAST 'label' < fixed-effect values ... >  
    < | random-effect values ... > , ... < / options > ;  
  ESTIMATE 'label' < fixed-effect values ... >  
    < | random-effect values ... >< / options > ;  
  LSMEANS fixed-effects < / options > ;
```

email

MSE? Who would want MSE? The MIXED model approach gives a maximum likelihood estimate, not a least squares estimate. Sums of squares are not calculated, and aren't needed. These are replaced by the variance component estimates.

However, in recognition that the occasional dinosaur of a referee will ask for MSE, or some other least squares related estimate, SAS has provided a mechanism (with certain restrictions). IF, and only IF, the variance component model has no SUBJECT= effects and IF there is no REPEATED statement, then you can request METHOD=TYPE3 (or TYPE1, or TYPE2, whatever) and the results will mimic the old GLM procedure from the last millennium. Note that for some analyses the standard errors computed for ESTIMATE and LSMEANS statements will not be correct (one reason most of us have abandoned the archaic GLM procedure).