SAS Education

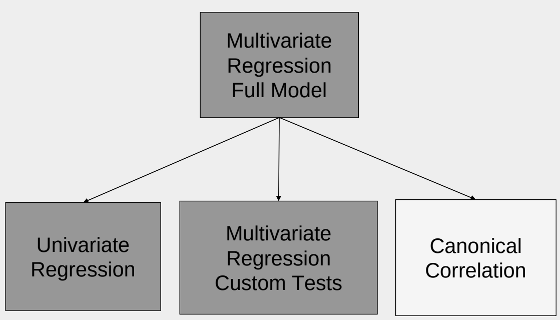
Section 3.3

Canonical Correlation

Objectives

- Define canonical correlation.
- Identify the relationship between canonical correlation analysis and multivariate regression.
- Relate eigenvalues to canonical correlation.
- Perform canonical variate analysis using PROC REG and PROC CANCORR.
- Interpret canonical correlation output, test for significance of canonical variates, and save out canonical scores.

Interpreting Multivariate Regression



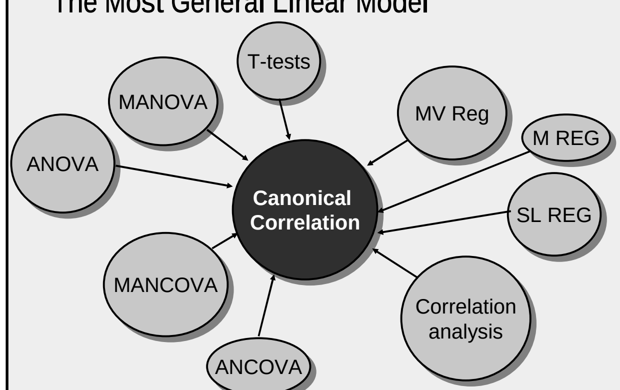
The Connection: R^2 and Eigenvalues

- Just as $\mathbf{HE}^{-1}$ is related to the *F*-ratio for a univariate model, $\mathbf{HT}^{-1}$ is related to R^2.
- Recall that R^2 is the square of the multiple correlation between the set of Xs and Y in a linear model. It is computed as SS_B/SS_T.
- In a multivariate linear model, the eigenvalues of $\mathbf{HT}^{-1}$ are a multivariate generalization of the R^2 in univariate models.
- The eigenvalues of $\mathbf{HT}^{-1}$ are the square of the canonical correlation coefficients.

More on the Eigenvalues of HT^{-1}

- There are *min(p,k)* nonzero eigenvalues in multivariate regression.
- The eigenvalues of $\mathbf{HT}^{-1}$ are equal to the square of the canonical correlations.
- Therefore, the square root of each eigenvalue is a canonical correlation.
- There are *min(p,k)* canonical correlations that can be computed for a data set.
- In the univariate case, *min(p,k)* = 1, therefore, there is always one value for R^2.

The Most General Linear Model



Canonical Correlation in PROC REG

You can use canonical correlation to understand multivariate multiple regression output.

- In PROC REG, you can use the CANPRINT option with the MTEST statement.
- You will see canonical correlations, likelihood ratio significance tests, and information about **HT**$^{-1}$ and **HE**$^{-1}$.

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Canonical Correlation Using PROC REG

ch3s3d1.sas

This demonstration illustrates how to obtain information about canonical correlation in PROC REG

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What Have You Learned?

- Of the two possible dimensions of association between the predictors and the responses, only one of them is statistically significant.
- One canonical correlation is significantly different from zero, the other is not.

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More Detailed Analysis

- The REG procedure provides canonical correlation information from a multivariate multiple regression.
- You may want to investigate the canonical correlations in greater detail.
- The CANCORR procedure provides a great deal of useful information and statistics for canonical correlation analysis.
- Canonical correlation may provide more useful and interesting information than multivariate regression analysis.

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What Is Really Going On?

From variable sets **X** and **Y**, find the linear combination of each set of variables that is most highly correlated with a linear combination of the other set. These combinations are the first canonical variates:

$$w_1 = a_1 x_1 + a_2 x_2 + ... + a_k x_k$$
$$v_1 = b_1 y_1 + b_2 y_2 + ... + b_p y_p$$
$$r_{w_1 v_1} = \text{ first canonical correlation}$$

Repeat this for a second pair of variates, uncorrelated with the first set.

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For Example…

Look at the X and Y matrices below:

$$\begin{array}{cc} Y_1\ Y_2\ Y_3\ Y_4 & X_1\ X_2\ X_3 \\ \begin{bmatrix} 2 & 3 & 5 & 8 \\ 3 & 4 & 7 & 7 \\ 2 & 4 & 5 & 7 \\ 1 & 3 & 4 & 6 \\ 2 & 4 & 5 & 9 \\ 4 & 3 & 6 & 6 \\ 4 & 6 & 4 & 7 \\ 2 & 5 & 6 & 7 \end{bmatrix} & \begin{bmatrix} 5 & 2 & 3 \\ 4 & 1 & 4 \\ 6 & 4 & 4 \\ 7 & 6 & 5 \\ 5 & 4 & 3 \\ 5 & 2 & 5 \\ 5 & 5 & 5 \\ 8 & 5 & 4 \end{bmatrix} \end{array}$$

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For Example…

The best set of coefficients, from canonical correlation analysis:

$$w_1 = .34x_1 - .49x_2 + 1.22x_3$$

$$v_1 = .33y_1 - .11y_2 + .17y_3 - .83y_4$$

$$r_{v_1 w_1} = .99$$

- v_1 and w_1 make up the first pair of canonical variates.
- min(p,k), = 3 pairs of variates can be computed for this data set.

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Canonical Correlation Using PROC CANCORR

In addition to the canonical correlations, you see the following:

- raw and standardized canonical coefficients for each canonical variate
- simple correlations for all variable and variate combinations
- step-down likelihood-ratio tests for canonical correlations
- multivariate tests for H_0:all canonical correlations = 0
- adjusted canonical correlations and standard errors of the canonical correlations.

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Likelihood-ratio Tests of Canonical Variates

The likelihood-ratio test of $\mathbf{HE}^{-1}$ is equal to Wilks' Lambda and can be converted to an approximate F test.

- This is a step-down method.
- The null hypothesis is that the canonical correlation $\mathbf{R}_i$ and all those smaller than $\mathbf{R}_i$ = 0.
- This test is performed sequentially for all canonical variate pairs.

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Interpreting the Canonical Correlation

Canonical coefficients are just like regression coefficients:

- They depend on the other variables in the model.
- They depend on the scale of measurement.
- Standardized coefficients address the scaling issue, but they do not address the problem of dependencies among variables.
- Coefficients are useful for prediction but not for interpretation.

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Interpreting the Canonical Correlation

A more useful way to interpret the canonical correlation in terms of the input variables is to look at the simple correlation statistics. For each pair of variates, look at

- the correlation between each variable and its canonical variate
- the correlation between each variable and the canonical variate for the other set of variables.

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Assumptions of Canonical Correlation

Goal: Description

- Minimal measurement error
- Unrestricted variances among variables
- Similar distributional shapes among variables

Goal: Inference

- Assumptions for Description, and at least one set of variables follows a multivariate normal distribution

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The CANCORR Procedure

General form of the CANCORR procedure:

```
PROC CANCORR <options>;
      <VAR v-variables;>
      WITH w-variables;
RUN;
```

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Pathological Gambling Example

Two questionnaires:

12 Items:
DSM-IV Criteria

↔

20 Items:
Gamblers Anonymous

Research questions:

- Are they measuring the same dimensions?
- How many dimensions?

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Canonical Correlation Analysis with PROC CANCORR

ch3s3d2.sas

This demonstration illustrates the CANCORR procedure for canonical correlation analysis

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What Have You Learned?

- Of the twelve canonical correlations, four are significantly greater than zero.
- Each pair of canonical variates has meaningful interpretation.
- The variables that are most correlated with their own variates are also most correlated with the corresponding variate from the other set of variables.

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Exercises

This exercise reinforces the concepts discussed previously.

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