Random coefficients model

$Y_{ij} = \beta_0 + s_i + (\beta_1 + d_i)X_{ij} + e_{ij}$

Note that

the β_k are fixed estimates of the population average
the dependent variable is Y_{ij}
X_{ij} is the independent variable
e_{ij} the residual term
s_i is the random effect of subjects on the intercept
d_i is the random effect of subjects on the slope

The random components are assumed to be NIDrv(0, σ^2) where the variances are σ_s^2 for s_i, σ_d^2 for d_i and σ^2 for e_{ij}

In a drug experiment where there are “i” subjects and “j” doses of a drug the new terms with the “i” subscript allow estimating the response for a particular patient.
So there are two levels of interest

$\beta_0 + \beta_1X_{ij}$	gives the population average slope and intercept
$\beta_0 + s_i + (\beta_1 + d_i)X_{ij}$	gives the values for a particular patient

These are loosely related to the concepts of “broad inference space” and “narrow inference space”. Broad inference space refers to inference made about average performance made across many categories (e.g. all possible subjects under a wide range of conditions) while narrow inference space defines a smaller subset of conditions of interest.

The original model
$Y_{ij} = \beta_0 + s_i + (\beta_1 + d_i)X_{ij} + e_{ij}$

Can also be expressed as a somewhat more familiar form

$Y_{ij} = \beta_0 + \beta_1X_{ij} + e^*_{ij}$

where $e^*_{ij} = s_i + d_i + e_{ij}$

Example 1

This study of exercise therapy has three between-subjects treatment levels; a control group that did not lift weights, in group RI the weightlifting repetitions were increases drui9ng the study and in group WI the amount of weight was increased. The repeated measure was time, the within subject measure. There were a total of 57 subjects distributed unequally among the treatments.

Example 2

The experiment was conducted on ten randomly selected varieties of winter wheat selected for tolerance to dry conditions. Each variety was randomly assigned to 6 one acre plots. Pre-plant moisture (top 36 inches of soil) was thought to be an influencing factor. Moisture was measured on each plot and was treated as a covariable. The fixed effects were the intercept and slope, which estimate the population intercept and slope for each variety.

Example 3

A botanist is studying the growth pattern in orange trees. The data shows a slightly nonlinear trend with an increase in variability over time. A logistic growth model was fitted. The objective is to fit an appropriate model while addressing intersubject variability and heterogeneity.