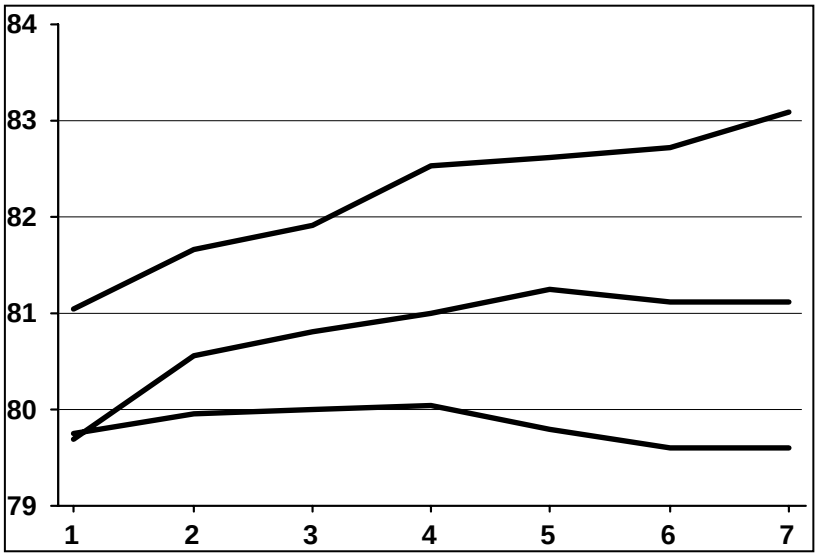
```
1          /*-------------------------------------------------------------
2            SAS System for Mixed Models (1996)
3            by Ramon C. Littell, Ph.D., George A. Milliken, Ph.D.,
4            Walter W. Stroup, Ph.D., and Russell D. Wolfinger, Ph.D.(page 87 ff)
5
6            SAS Publications order # 55235
7            ISBN 1-55544-779-1
8            Copyright 1996 by SAS Institute Inc., Cary, NC, USA
9
10           This file contains the SAS code needed to produce the output
11           in this book with Release 6.11 of the SAS System.
12         -------------------------------------------------------------*/
13
14         /* Updated: 12JUN02 */
15         /* Modifications by Geaghan 14Nov02 */
16
17         ODS HTML style=minimal rs=none
18             body='C:\Geaghan\EXST\EXST7034New\Fall2002\SAS\Datasets Mixed
18        ! Model\MixModProgCh3.html' ;
NOTE: Writing HTML Body file: C:\Geaghan\EXST\EXST7034New\Fall2002\SAS\MixModProgCh3.html
19
20         options ps=256 ls=100 nocenter nodate nonumber;
21         title1 'Random Coeff Reg - SAS System for Mixed Models, 1996, Littell, et al.';
22         data weights;
23            input subj program$ s1 s2 s3 s4 s5 s6 s7;
24         datalines;
NOTE: The data set WORK.WEIGHTS has 57 observations and 9 variables.
NOTE: DATA statement used:
      real time           0.04 seconds
82         ;
83
84         data weight2;
85            set weights;
86            time=1; strength=s1; output;
87            time=2; strength=s2; output;
88            time=3; strength=s3; output;
89            time=4; strength=s4; output;
90            time=5; strength=s5; output;
91            time=6; strength=s6; output;
92            time=7; strength=s7; output;
93         *    label subj = subject (experimental unit);
94         *    label program = weight lifting program;
95            keep subj program time strength;
96         run;
NOTE: There were 57 observations read from the data set WORK.WEIGHTS.
NOTE: The data set WORK.WEIGHT2 has 399 observations and 4 variables.
NOTE: DATA statement used:
      real time           0.05 seconds
97
98
99         data weight2; set weight2; t=time; tt=t*t; run;
NOTE: There were 399 observations read from the data set WORK.WEIGHT2.
NOTE: The data set WORK.WEIGHT2 has 399 observations and 6 variables.
NOTE: DATA statement used:
      real time           0.00 seconds
100
101        title2 'ANCOVA without Random coefficients';
102        proc mixed data=weight2; class program subj;
103        model strength = program time time*program time*time time*time*program / htype=1 3;
104        run;
NOTE: The PROCEDURE MIXED printed page 1.
NOTE: PROCEDURE MIXED used:
      real time           0.11 seconds
```

```
Random Coeff Reg - SAS System for Mixed Models, 1996, Littell, et al.
ANCOVA without Random coefficients

The Mixed Procedure

Model Information
Data Set                     WORK.WEIGHT2
Dependent Variable           strength
Covariance Structure         Diagonal
Estimation Method            REML
Residual Variance Method     Profile
Fixed Effects SE Method      Model-Based
Degrees of Freedom Method    Residual

Class Level Information
Class      Levels    Values
program         3    CONT RI WI
subj           21    1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21

Dimensions
Covariance Parameters             1
Columns in X                     12
Columns in Z                      0
Subjects                          1
Max Obs Per Subject             399
Observations Used               399
Observations Not Used             0
Total Observations              399

Covariance Parameter Estimates
Cov Parm     Estimate
Residual      10.4770

Fit Statistics
-2 Res Log Likelihood          2078.5
AIC (smaller is better)        2080.5
AICC (smaller is better)       2080.5
BIC (smaller is better)        2084.5
```




```
Type 1 Tests of Fixed Effects
Effect             Num DF  Den DF    F Value    Pr > F
program                 2     390      20.02    <.0001
time                    1     390       3.89    0.0493
time*program            2     390       1.93    0.1469
time*time               1     390       0.94    0.3332
time*time*program       2     390       0.07    0.9343

Type 3 Tests of Fixed Effects
Effect             Num DF  Den DF    F Value    Pr > F
program                 2     390       0.39    0.6749
time                    1     390       1.95    0.1629
time*program            2     390       0.19    0.8311
time*time               1     390       1.01    0.3156
time*time*program       2     390       0.07    0.9343

106        title2 'Random coefficient Regression';
107        proc mixed data=weight2 covtest; class program subj;
108        model strength = program time time*program time*time time*time*program / htype=1 3;
109           random subj(program) time*subj(program) time*time*subj(program);
110        run;
NOTE: Convergence criteria met.
NOTE: The PROCEDURE MIXED printed page 2.
NOTE: PROCEDURE MIXED used:
      real time           47.88 seconds
```

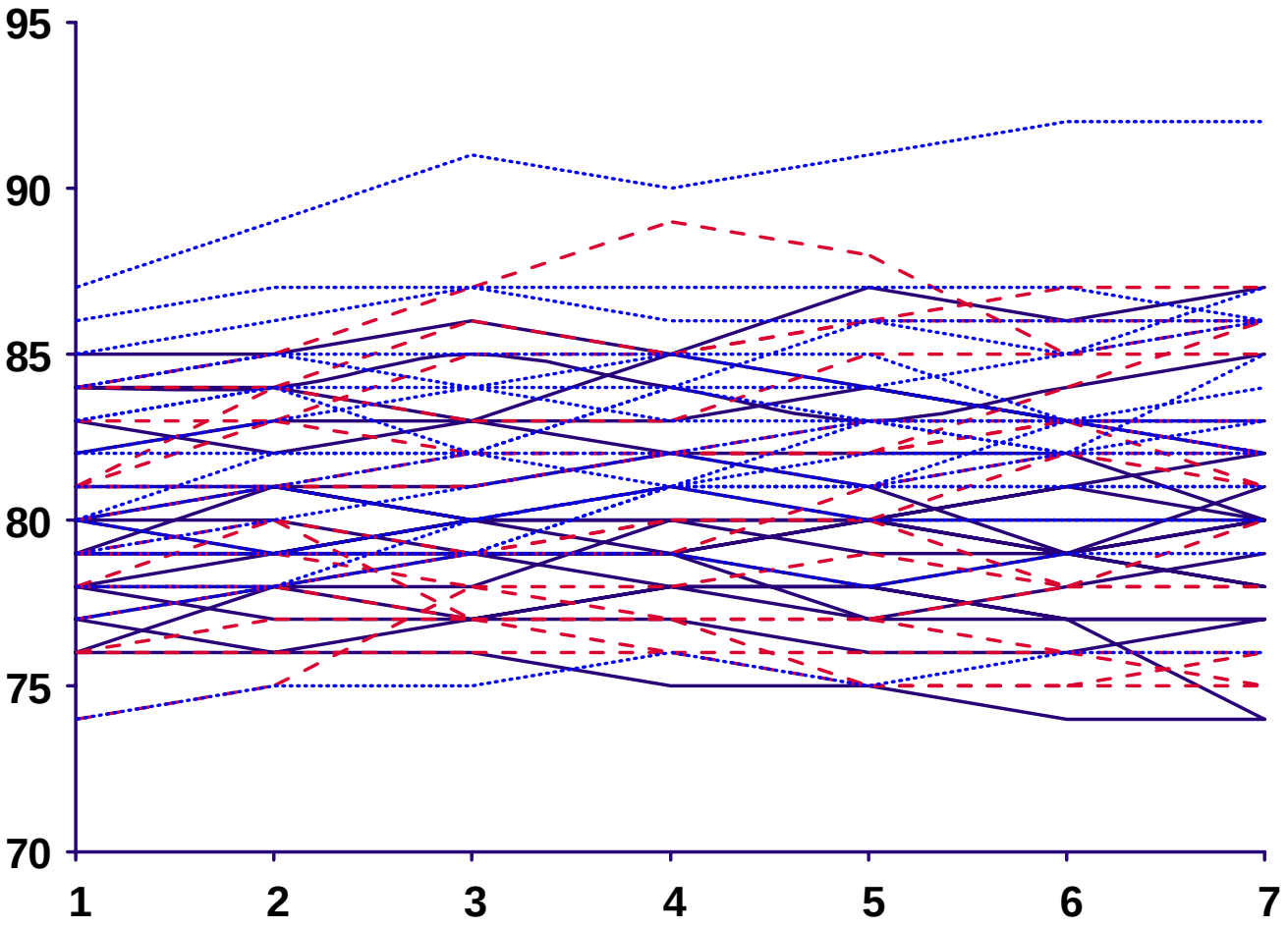
```
Random Coeff Reg - SAS System for Mixed Models, 1996, Littell, et al.
Random coefficient Regression

The Mixed Procedure

Model Information
Data Set                     WORK.WEIGHT2
Dependent Variable           strength
Covariance Structure         Variance Components
Subject Effect               subj
Estimation Method            REML
Residual Variance Method     Profile
Fixed Effects SE Method      Model-Based
Degrees of Freedom Method    Containment

Class Level Information
Class      Levels    Values
program         3    CONT RI WI
subj           21    1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21

Dimensions
Covariance Parameters             4
Columns in X                     12
Columns in Z                    171
Subjects                          1
Max Obs Per Subject             399
Observations Used               399
Observations Not Used             0
Total Observations              399

Iteration History
Iteration    Evaluations    -2 Res Log Like       Criterion
        0              1      2078.51675124
        1              2      1319.94926364      0.00485216
        2              1      1318.24781059      0.00046044
        3              1      1318.09966078      0.00000633
        4              1      1318.09773832      0.00000000
Convergence criteria met.

Covariance Parameter Estimates
                                    Standard         Z
Cov Parm                Estimate       Error     Value        Pr Z
subj(program)             8.8508      1.7765      4.98      <.0001
time*subj(program)       0.09515     0.03191      2.98      0.0014
time*time*subj(prog)    0.000609    0.000439      1.39      0.0825
Residual                  0.5834     0.05100     11.44      <.0001

          Fit Statistics
-2 Res Log Likelihood          1318.1
AIC (smaller is better)        1326.1
AICC (smaller is better)       1326.2
BIC (smaller is better)        1334.3

           Type 1 Tests of Fixed Effects
Effect              Num DF  Den DF    F Value    Pr > F
program                  2      54       2.80    0.0697
time                     1      54       9.39    0.0034
time*program             2      54       4.65    0.0136
time*time                1      54      15.50    0.0002
time*time*program        2      54       1.12    0.3330

           Type 3 Tests of Fixed Effects
Effect              Num DF  Den DF    F Value    Pr > F
program                  2      54       0.98    0.3836
time                     1      54      29.15    <.0001
time*program             2      54       2.76    0.0723
time*time                1      54      16.67    0.0001
time*time*program        2      54       1.12    0.3330
```

```
112        title2 'Random coefficient Regression - the old way';
113        proc sort data=weight2; by program subj time; run;
NOTE: There were 399 observations read from the data set WORK.WEIGHT2.
NOTE: The data set WORK.WEIGHT2 has 399 observations and 6 variables.
NOTE: PROCEDURE SORT used:
      real time           0.11 seconds
114        proc reg data=weight2 noprint outest=next1; by program subj;
115            model strength = t tt;
116        run;
NOTE: 7 observations read.
NOTE: 7 observations used in computations.
NOTE: The above message was for the following by-group:
      program=CONT subj=1
NOTE: 7 observations read.
NOTE: 7 observations used in computations.
NOTE: The above message was for the following by-group:
      program=CONT subj=2
. . .
      program=WI subj=20
NOTE: 7 observations read.
NOTE: 7 observations used in computations.
NOTE: The above message was for the following by-group:
      program=WI subj=21
NOTE: Interactivity disabled with BY processing.
NOTE: PROCEDURE REG used:
      real time           0.17 seconds
NOTE: The data set WORK.NEXT1 has 57 observations and 10 variables.
117        proc print data=next1; run;

NOTE: There were 57 observations read from the data set WORK.NEXT1.
NOTE: The PROCEDURE PRINT printed page 3.
NOTE: PROCEDURE PRINT used:
      real time           0.10 seconds
```

Random Coeff Reg - SAS System for Mixed Models, 1996, Littell, et al.
Random coefficient Regression - the old way

Obs	program	subj	_MODEL_	_TYPE_	_DEPVAR_	_RMSE_	Intercept	t	tt	strength
1	CONT	1	MODEL1	PARMS	strength	0.69864	84.7143	0.22619	0.01190	-1
2	CONT	2	MODEL1	PARMS	strength	0.52327	80.7143	-0.91667	0.08333	-1
3	CONT	3	MODEL1	PARMS	strength	0.44987	78.8571	-0.97619	0.09524	-1
4	CONT	4	MODEL1	PARMS	strength	0.79433	84.7143	-0.44048	0.05952	-1
5	CONT	5	MODEL1	PARMS	strength	0.68139	81.0000	-0.46429	0.03571	-1
6	CONT	6	MODEL1	PARMS	strength	0.77919	74.0000	2.32143	-0.32143	-1
7	CONT	7	MODEL1	PARMS	strength	0.74801	79.2857	-0.16667	0.04762	-1
8	CONT	8	MODEL1	PARMS	strength	0.32733	76.2857	-0.10714	-0.03571	-1
9	CONT	9	MODEL1	PARMS	strength	0.62678	75.4286	1.46429	-0.10714	-1
10	CONT	10	MODEL1	PARMS	strength	0.82375	80.0000	-0.71429	0.07143	-1
11	CONT	11	MODEL1	PARMS	strength	0.32733	82.4286	-1.32143	0.17857	-1
12	CONT	12	MODEL1	PARMS	strength	0.64550	76.1429	0.45238	-0.04762	-1
13	CONT	13	MODEL1	PARMS	strength	0.40825	81.2857	0.90476	-0.09524	-1
14	CONT	14	MODEL1	PARMS	strength	0.30861	84.4286	-0.11905	-0.11905	-1
15	CONT	15	MODEL1	PARMS	strength	0.48795	77.0000	2.30952	-0.26190	-1
16	CONT	16	MODEL1	PARMS	strength	0.50000	80.5714	-1.35714	0.14286	-1
17	CONT	17	MODEL1	PARMS	strength	0.97590	80.8571	1.52381	-0.19048	-1
18	CONT	18	MODEL1	PARMS	strength	0.50000	77.1429	0.92857	-0.14286	-1
19	CONT	19	MODEL1	PARMS	strength	0.44987	80.5714	-0.63095	0.08333	-1
20	CONT	20	MODEL1	PARMS	strength	0.72375	76.7143	1.54762	-0.16667	-1
21	RI	1	MODEL1	PARMS	strength	0.89974	78.7143	0.26190	-0.02381	-1
22	RI	2	MODEL1	PARMS	strength	0.50000	81.7143	1.03571	-0.03571	-1
23	RI	3	MODEL1	PARMS	strength	0.72375	80.5714	0.90476	-0.09524	-1
24	RI	4	MODEL1	PARMS	strength	0.74801	79.8571	0.84524	-0.08333	-1
25	RI	5	MODEL1	PARMS	strength	0.69864	80.4286	-0.09524	0.11905	-1
26	RI	6	MODEL1	PARMS	strength	0.24398	75.5714	0.36905	-0.05952	-1
27	RI	7	MODEL1	PARMS	strength	0.98198	80.5714	1.14286	-0.07143	-1
28	RI	8	MODEL1	PARMS	strength	0.70711	75.5714	1.35714	-0.07143	-1
29	RI	9	MODEL1	PARMS	strength	1.24881	80.8571	3.20238	-0.36905	-1
30	RI	10	MODEL1	PARMS	strength	0.64550	71.1429	2.86905	-0.27381	-1

31	RI	11	MODEL1	PARMS	strength	0.30861	75.4286	0.88095	-0.11905	-1
32	RI	12	MODEL1	PARMS	strength	0.59761	83.0000	0.92857	-0.07143	-1
33	RI	13	MODEL1	PARMS	strength	0.64550	79.4286	-0.26190	0.09524	-1
34	RI	14	MODEL1	PARMS	strength	0.61721	79.8571	-1.45238	0.11905	-1
35	RI	15	MODEL1	PARMS	strength	1.19024	79.8571	-0.84524	0.01190	-1
36	RI	16	MODEL1	PARMS	strength	0.36187	82.7143	1.54762	-0.23810	-1
37	WI	1	MODEL1	PARMS	strength	0.69864	85.4286	-0.84524	0.08333	-1
38	WI	2	MODEL1	PARMS	strength	0.48795	73.5714	0.66667	-0.04762	-1
39	WI	3	MODEL1	PARMS	strength	1.08562	84.0000	-0.71429	0.07143	-1
40	WI	4	MODEL1	PARMS	strength	0.24398	85.2857	0.95238	-0.11905	-1
41	WI	5	MODEL1	PARMS	strength	0.61721	81.2857	0.95238	-0.04762	-1
42	WI	6	MODEL1	PARMS	strength	0.61721	79.4286	-0.11905	0.02381	-1
43	WI	7	MODEL1	PARMS	strength	0.65465	78.4286	0.21429	0.07143	-1
44	WI	8	MODEL1	PARMS	strength	0.74801	85.7143	1.79762	-0.13095	-1
45	WI	9	MODEL1	PARMS	strength	0.32733	80.7143	0.10714	0.03571	-1
46	WI	10	MODEL1	PARMS	strength	0.91937	81.1429	0.41667	0.05952	-1
47	WI	11	MODEL1	PARMS	strength	0.36187	77.7143	1.05952	-0.08333	-1
48	WI	12	MODEL1	PARMS	strength	0.42258	77.0000	1.96429	-0.17857	-1
49	WI	13	MODEL1	PARMS	strength	0.30861	82.4286	0.88095	-0.11905	-1
50	WI	14	MODEL1	PARMS	strength	1.19024	80.2857	0.63095	-0.01190	-1
51	WI	15	MODEL1	PARMS	strength	0.52327	77.7143	0.33333	-0.02381	-1
52	WI	16	MODEL1	PARMS	strength	0.87966	82.2857	0.15476	0.01190	-1
53	WI	17	MODEL1	PARMS	strength	0.79433	79.2857	0.20238	-0.01190	-1
54	WI	18	MODEL1	PARMS	strength	0.65465	79.5714	1.14286	-0.14286	-1
55	WI	19	MODEL1	PARMS	strength	0.52327	84.5714	0.83333	-0.09524	-1
56	WI	20	MODEL1	PARMS	strength	0.32733	74.5714	2.32143	-0.17857	-1
57	WI	21	MODEL1	PARMS	strength	0.48795	80.5714	-0.33333	0.09524	-1



Estimates from PROC MIXED LSMEANS statements for individually fitted models. The analyses below are for the individually fitted intercepts and slopes. Note that the tests are essentially "TYPE III" tests.

```
120        proc mixed data=next1; classes program;
121           title3 'Analysis of intercept';
122           model intercept = program;
123           lsmeans program;
124        run;
NOTE: The PROCEDURE MIXED printed page 4.
NOTE: PROCEDURE MIXED used:      real time           0.12 seconds
      cpu time            0.07 seconds

Type 3 Tests of Fixed Effects
Effect     Num DF  Den DF    F Value    Pr > F
program         2      54       0.96    0.3901

Least Squares Means               Standard
Effect     program    Estimate       Error      DF    t Value    Pr > |t|
program    CONT        79.6071      0.7231      54     110.10      <.0001
program    RI          79.0804      0.8084      54      97.82      <.0001
program    WI          80.5238      0.7056      54     114.11      <.0001


125        proc mixed data=next1; classes program;
126           title3 'Analysis of linear coefficient';
127           model t = program;
128           lsmeans program;
129        run;
NOTE: The PROCEDURE MIXED printed page 5.
NOTE: PROCEDURE MIXED used:      real time           0.12 seconds
      cpu time            0.07 seconds

Type 3 Tests of Fixed Effects
Effect     Num DF  Den DF    F Value    Pr > F
program         2      54       1.40    0.2563

Least Squares Means               Standard
Effect     program    Estimate       Error      DF    t Value    Pr > |t|
program    CONT          0.2232      0.2353      54       0.95      0.3471
program    RI            0.7932      0.2631      54       3.01      0.0039
program    WI            0.6009      0.2296      54       2.62      0.0115


130        proc mixed data=next1; classes program;
131           title3 'Analysis of quadratic coefficient';
132           model tt = program;
133           lsmeans program;
134        run;
NOTE: The PROCEDURE MIXED printed page 6.
NOTE: PROCEDURE MIXED used:      real time           0.18 seconds
      cpu time            0.08 seconds

Type 3 Tests of Fixed Effects
Effect     Num DF  Den DF    F Value    Pr > F
program         2      54       0.58    0.5626

Least Squares Means               Standard
Effect     program    Estimate       Error      DF    t Value    Pr > |t|
program    CONT       -0.03393     0.02700      54      -1.26      0.2143
program    RI         -0.07292     0.03019      54      -2.42      0.0191
program    WI         -0.03515     0.02635      54      -1.33      0.1878
```

Means model estimates of slopes and intercepts and LSMeans results from the PROC MIXED analysis of the individual slope and intercept estimates.

Estimates from MEANS MODEL for ordinary least squares model.

$$Y_{ij} = \beta_0 + \beta_1 X_{ij} + e_{ij}$$

```
142        title3 'ANCOVA without Random coefficients - Means model';
143        proc mixed data=weight2; class program subj;
144           model strength = program time*program time*time*program / htype=1 3 NOINT SOLUTION;
145        run;
NOTE: The PROCEDURE MIXED printed page 10.
NOTE: PROCEDURE MIXED used:
      real time           0.14 seconds
      cpu time            0.09 seconds

Covariance Parameter Estimates
Cov Parm     Estimate
Residual      10.4770

Solution for Fixed Effects
                                          Standard
Effect               program    Estimate       Error      DF    t Value    Pr > |t|
program              CONT        79.6071      1.1279     390      70.58      <.0001
program              RI          79.0804      1.2611     390      62.71      <.0001
program              WI          80.5238      1.1007     390      73.15      <.0001
time*program         CONT         0.2232      0.6464     390       0.35      0.7300
time*program         RI           0.7932      0.7227     390       1.10      0.2731
time*program         WI           0.6009      0.6308     390       0.95      0.3414
time*time*program    CONT       -0.03393     0.07897     390      -0.43      0.6677
time*time*program    RI         -0.07292     0.08829     390      -0.83      0.4094
time*time*program    WI         -0.03515     0.07707     390      -0.46      0.6486
```

Estimates from MEANS MODEL for random coefficients model.

$$Y_{ij} = \beta_0 + s_i + (\beta_1 + d_i)X_{ij} + e_{ij}$$

```
147        title3 'Random coefficient Regression - means model';
148        proc mixed data=weight2 covtest; class program subj;
149           model strength = program time*program time*time*program / htype=1 3 NOINT SOLUTION;
150           random subj(program) time*subj(program) time*time*subj(program);
151        run;
NOTE: Convergence criteria met.
NOTE: The PROCEDURE MIXED printed page 11.
NOTE: PROCEDURE MIXED used:
      real time           0.24 seconds
       cpu time              0.18 seconds

Covariance Parameter Estimates
                                  Standard         Z
Cov Parm                Estimate       Error     Value        Pr Z
subj(program)             8.8508      1.7765      4.98      <.0001
time*subj(program)       0.09515     0.03191      2.98      0.0014
time*time*subj(prog)    0.000609    0.000439      1.39      0.0825
Residual                  0.5834     0.05100     11.44      <.0001

Solution for Fixed Effects
                                          Standard
Effect               program    Estimate       Error      DF    t Value    Pr > |t|
program              CONT        79.6071      0.7229      54     110.12      <.0001
program              RI          79.0804      0.8082      54      97.85      <.0001
program              WI          80.5238      0.7055      54     114.14      <.0001
time*program         CONT         0.2232      0.1736      54       1.29      0.2039
time*program         RI           0.7932      0.1941      54       4.09      0.0001
time*program         WI           0.6009      0.1694      54       3.55      0.0008
time*time*program    CONT       -0.03393     0.01894      54      -1.79      0.0788
time*time*program    RI         -0.07292     0.02117      54      -3.44      0.0011
time*time*program    WI         -0.03515     0.01848      54      -1.90      0.0626
```

Selected PROC UNIVARIATE results.

```
132        proc univariate data=next1 plot normal;
133           title3 'Proc univariate analysis of coefficients';
134           var intercept t tt;
135        run;
NOTE: Unable to find the "Note" style element. Default style attributes will be used.
NOTE: The PROCEDURE UNIVARIATE printed pages 7-9.
NOTE: PROCEDURE UNIVARIATE used:
      real time           0.04 seconds

The UNIVARIATE Procedure
Variable:  Intercept  (Intercept)

Moments
N                          57    Sum Weights                57
Mean                79.7969925    Sum Observations    4548.42857
Std Deviation       3.23124183    Variance            10.4409237
Skewness            -0.2995726    Kurtosis            -0.0796669
Uncorrected SS      363535.612    Corrected SS        584.691729
Coeff Variation     4.04932783    Std Error Mean      0.42798844

Tests for Normality
Test                   --Statistic---    -----p Value------
Shapiro-Wilk           W     0.975724    Pr < W      0.3058
Kolmogorov-Smirnov     D     0.103805    Pr > D      0.1280
Cramer-von Mises       W-Sq  0.094039    Pr > W-Sq   0.1351
Anderson-Darling       A-Sq  0.498705    Pr > A-Sq   0.2109

The UNIVARIATE Procedure
Variable:  t

Moments
N                          57    Sum Weights                57
Mean                0.52234754    Sum Observations    29.7738095
Std Deviation       1.05975803    Variance            1.12308709
Skewness            0.28897829    Kurtosis            -0.1595764
Uncorrected SS      78.4451531    Corrected SS         62.892877
Coeff Variation     202.883705    Std Error Mean      0.14036838

Tests for Normality
Test                   --Statistic---    -----p Value------
Shapiro-Wilk           W     0.982471    Pr < W      0.5761
Kolmogorov-Smirnov     D     0.071551    Pr > D     >0.1500
Cramer-von Mises       W-Sq  0.040028    Pr > W-Sq  >0.2500
Anderson-Darling       A-Sq  0.268092    Pr > A-Sq  >0.2500

The UNIVARIATE Procedure
Variable:  tt

Moments
N                          57    Sum Weights                57
Mean               -0.0453216    Sum Observations    -2.5833333
Std Deviation       0.11984086    Variance            0.01436183
Skewness            -0.5079757    Kurtosis            0.01462346
Uncorrected SS      0.92134354    Corrected SS        0.80426264
Coeff Variation     -264.42307    Std Error Mean      0.01587331

Tests for Normality
Test                   --Statistic---    -----p Value------
Shapiro-Wilk           W     0.973544    Pr < W      0.2442
Kolmogorov-Smirnov     D     0.072336    Pr > D     >0.1500
Cramer-von Mises       W-Sq   0.04898    Pr > W-Sq  >0.2500
Anderson-Darling       A-Sq  0.398514    Pr > A-Sq  >0.2500
```

```
1          /*-------------------------------------------------------------
2            SAS System for Mixed Models (1996)
3            by Ramon C. Littell, Ph.D., George A. Milliken, Ph.D.,
4            Walter W. Stroup, Ph.D., and Russell D. Wolfinger, Ph.D. (page 256 ff)
5
6            SAS Publications order # 55235
7            ISBN 1-55544-779-1
8            Copyright 1996 by SAS Institute Inc., Cary, NC, USA
9
10           This file contains the SAS code needed to produce the output
11           in this book with Release 6.11 of the SAS System.
12         -------------------------------------------------------------*/
13
14         /* Updated: 12JUN02 */
15         /* Modifications by Geaghan 14Nov02 */
6
17         ODS HTML style=minimal rs=none
18             body='C:\Geaghan\EXST\EXST7034New\Fall2002\Datasets Mixed Model\Rand Coeff Winter Wheat Example.html';
NOTE: Writing HTML Body file: C:\Geaghan\EXST\EXST7034New\Fall2002\Datasets Mixed Model\Rand
      Coeff Winter Wheat Example.html
19
20         options ps=256 ls=100 nocenter nodate nonumber;
21         title1 'Random Coefficients - from SAS System for Mixed Models, 1996, Littell, et al.';
22
23         data wheat;
24            input id variety yield moist;
25            datalines;
NOTE: The data set WORK.WHEAT has 60 observations and 4 variables.
NOTE: DATA statement used:
      real time           0.17 seconds
86         ;
89         proc mixed covtest; class variety;
90            title2 'Random Coefficients - Winter Wheat Example - Unstructured Covariance';
91            model yield = moist / solution;
92            random int moist / type=un sub=variety solution;
93         run;
NOTE: Convergence criteria met.
NOTE: The PROCEDURE MIXED printed page 1.
NOTE: PROCEDURE MIXED used:
      real time           0.16 seconds

Random Coefficients - from SAS System for Mixed Models, 1996, Littell, et al.
Random Coefficients - Winter Wheat Example - Unstructured Covariance

The Mixed Procedure
                  Model Information
Data Set                      WORK.WHEAT
Dependent Variable            yield
Covariance Structure          Unstructured
Subject Effect                variety
Estimation Method             REML
Residual Variance Method      Profile
Fixed Effects SE Method       Model-Based
Degrees of Freedom Method     Containment

               Class Level Information
Class      Levels    Values
variety        10    1 2 3 4 5 6 7 8 9 10

            Dimensions
Covariance Parameters             4
Columns in X                      2
Columns in Z Per Subject          2
Subjects                         10
Max Obs Per Subject               6
Observations Used                60
Observations Not Used             0
Total Observations               60
```

Iteration History

Iteration	Evaluations	-2 Res Log Like	Criterion
0	1	336.66906201	
1	2	248.89397809	60752010.673
2	1	232.33494647	16486923.469
3	1	217.46129037	4107075.9894
4	1	205.31750341	912992.45620
5	1	196.20212291	189697.45638
6	2	190.49720530	0.13991880
7	1	189.10253837	0.03859915
8	1	187.07627790	0.01498753
9	1	186.33196655	0.00369168
10	1	186.16090609	0.00034166
11	1	186.14642918	0.00000385
12	1	186.14627489	0.00000000

Convergence criteria met.

Covariance Parameter Estimates

Cov Parm	Subject	Estimate	Standard Error	Z Value	Pr Z
UN(1,1)	variety	18.8942	9.1107	2.07	0.0190
UN(2,1)	variety	-0.07272	0.08242	-0.88	0.3776
UN(2,2)	variety	0.002394	0.001349	1.77	0.0380
Residual		0.3521	0.07902	4.46	<.0001

Fit Statistics

-2 Res Log Likelihood	186.1
AIC (smaller is better)	194.1
AICC (smaller is better)	194.9
BIC (smaller is better)	195.4

Null Model Likelihood Ratio Test

DF	Chi-Square	Pr > ChiSq
3	150.52	<.0001

Solution for Fixed Effects

Effect	Estimate	Standard Error	DF	t Value	Pr > |t|
Intercept	33.4339	1.3985	9	23.91	<.0001
moist	0.6617	0.01678	9	39.42	<.0001

Solution for Random Effects

Effect	variety	Estimate	Std Err Pred	DF	t Value	Pr > |t|
Intercept	1	0.9578	1.5101	40	0.63	0.5295
moist	1	-0.04921	0.02114	40	-2.33	0.0250
Intercept	2	-2.2843	1.5269	40	-1.50	0.1425
moist	2	-0.06670	0.02742	40	-2.43	0.0196
Intercept	3	-0.4081	1.5159	40	-0.27	0.7891
moist	3	0.06722	0.02231	40	3.01	0.0045
Intercept	4	0.6960	1.4819	40	0.47	0.6411
moist	4	-0.02331	0.02386	40	-0.98	0.3346
Intercept	5	1.1159	1.6929	40	0.66	0.5136
moist	5	-0.01990	0.02599	40	-0.77	0.4483
Intercept	6	4.6392	1.4671	40	3.16	0.0030
moist	6	0.02389	0.01989	40	1.20	0.2367
Intercept	7	-10.7300	1.4553	40	-7.37	<.0001
moist	7	0.05642	0.02220	40	2.54	0.0150
Intercept	8	2.4012	1.4842	40	1.62	0.1136
moist	8	0.02243	0.02213	40	1.01	0.3168
Intercept	9	-0.1762	1.5168	40	-0.12	0.9081
moist	9	0.02336	0.02227	40	1.05	0.3005
Intercept	10	3.7886	1.8340	40	2.07	0.0454
moist	10	-0.03421	0.02872	40	-1.19	0.2407

Type 3 Tests of Fixed Effects

Effect	Num DF	Den DF	F Value	Pr > F
moist	1	9	1554.14	<.0001

```
96         proc mixed covtest; class variety;
97            title2 'Random Coefficients - Winter Wheat Example - No Covariance';
98            model yield = moist / solution;
99            random int moist / sub=variety solution;
100        run;

NOTE: Convergence criteria met.
NOTE: The PROCEDURE MIXED printed page 2.
NOTE: PROCEDURE MIXED used:
      real time           0.05 seconds

Random Coefficients - from SAS System for Mixed Models, 1996, Littell, et al.
Random Coefficients - Winter Wheat Example - No Covariance

The Mixed Procedure

                  Model Information
Data Set                      WORK.WHEAT
Dependent Variable            yield
Covariance Structure          Variance Components
Subject Effect                variety
Estimation Method             REML
Residual Variance Method      Profile
Fixed Effects SE Method       Model-Based
Degrees of Freedom Method     Containment

              Class Level Information
Class      Levels    Values
variety        10    1 2 3 4 5 6 7 8 9 10

            Dimensions
Covariance Parameters             3
Columns in X                      2
Columns in Z Per Subject          2
Subjects                         10
Max Obs Per Subject               6
Observations Used                60
Observations Not Used             0
Total Observations               60

                    Iteration History
Iteration    Evaluations    -2 Res Log Like       Criterion
        0              1       336.66906201
        1              2       201.02887718      0.10681253
        2              1       194.28699351      0.07468370
        3              1       190.01509771      0.03894797
        4              1       187.95605215      0.01406786
        5              1       187.25879979      0.00294964
        6              1       187.12236564      0.00022009
        7              1       187.11302262      0.00000168
        8              1       187.11295447      0.00000000
Convergence criteria met.

                  Covariance Parameter Estimates
                                    Standard         Z
Cov Parm      Subject    Estimate       Error     Value        Pr Z
Intercept     variety     18.2825      8.7974      2.08      0.0188
moist         variety    0.002291    0.001299      1.76      0.0389
Residual                   0.3543     0.07996      4.43      <.0001

           Fit Statistics
-2 Res Log Likelihood           187.1
AIC (smaller is better)         193.1
AICC (smaller is better)        193.6
BIC (smaller is better)         194.0
```

Solution for Fixed Effects

Effect	Estimate	Standard Error	DF	t Value	Pr > |t|
Intercept	33.4108	1.3761	9	24.28	<.0001
moist	0.6619	0.01647	9	40.19	<.0001

Solution for Random Effects

Effect	variety	Estimate	Std Err Pred	DF	t Value	Pr > |t|
Intercept	1	0.9444	1.4870	40	0.64	0.5290
moist	1	-0.04866	0.02080	40	-2.34	0.0244
Intercept	2	-2.2272	1.5020	40	-1.48	0.1460
moist	2	-0.06849	0.02696	40	-2.54	0.0151
Intercept	3	-0.3504	1.4923	40	-0.23	0.8156
moist	3	0.06626	0.02195	40	3.02	0.0044
Intercept	4	0.7015	1.4589	40	0.48	0.6332
moist	4	-0.02294	0.02352	40	-0.97	0.3354
Intercept	5	1.0728	1.6596	40	0.65	0.5217
moist	5	-0.01874	0.02536	40	-0.74	0.4643
Intercept	6	4.6292	1.4448	40	3.20	0.0027
moist	6	0.02469	0.01958	40	1.26	0.2146
Intercept	7	-10.5860	1.4329	40	-7.39	<.0001
moist	7	0.05120	0.02191	40	2.34	0.0245
Intercept	8	2.3977	1.4613	40	1.64	0.1087
moist	8	0.02317	0.02180	40	1.06	0.2943
Intercept	9	-0.1414	1.4932	40	-0.09	0.9250
moist	9	0.02289	0.02191	40	1.04	0.3024
Intercept	10	3.5594	1.7867	40	1.99	0.0532
moist	10	-0.02940	0.02777	40	-1.06	0.2962

Type 3 Tests of Fixed Effects

Effect	Num DF	Den DF	F Value	Pr > F
moist	1	9	1615.30	<.0001

The difference in the two models is the inclusion of a term for the covariance. A test of the covariance parameter estimates indicated no difference. Another, better, test of the difference between the models is the likelihood ration test. This employs the –2 residual log likelihood from the "Fit statistics".

Full model (with covariance estimate)

Fit Statistics	
-2 Res Log Likelihood	186.1

Reduced model (without covariance estimate)

Fit Statistics	
-2 Res Log Likelihood	187.1

Difference 187.1 – 186.1 = 1 (this is a Chi square value)
Degrees of freedom difference in the models is = 1
P> = χ^2 = 0.317310813.
The test is not significant (α = 0.05)

```
1          /*-------------------------------------------------------------
2            SAS System for Mixed Models (1996)
3            by Ramon C. Littell, Ph.D., George A. Milliken, Ph.D.,
4            Walter W. Stroup, Ph.D., and Russell D. Wolfinger, Ph.D. (page 461 ff)
5
6            SAS Publications order # 55235
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9
10           This file contains the SAS code needed to produce the output
11           in this book with Release 6.11 of the SAS System.
12         -------------------------------------------------------------*/
13
14         /* Updated: 12JUN02 */
15         /* Modifications by Geaghan 2Dec02 */
16
17         ODS HTML style=minimal rs=none
18             body='C:\Geaghan\EXST\EXST7034New\Fall2002\Datasets Mixed Model\Chapt 12.html' ;
NOTE: Writing HTML Body file: C:\Geaghan\EXST\EXST7034New\Fall2002\Datasets Mixed Model\Chapt
      12.html
19
20         options ps=256 ls=100 nocenter nodate nonumber;
21         title1 'NLINMix - from SAS System for Mixed Models, 1996, Littell, et al.';
22
23         /*---NOTE: before running the NLINMIX code you must first
24              %include the NLINMIX macro.  That is, submit
25
26                 %inc 'nlinmix.sas' / nosource;
27
28              before running the code.---*/
29
30         %inc 'nlinmix.sas' / nosource;
1581
1582       data tree;
1583          input tree time x y;
1584          datalines;

NOTE: The data set WORK.TREE has 35 observations and 4 variables.
NOTE: DATA statement used:
      real time           0.11 seconds

1620       ;
1621
```

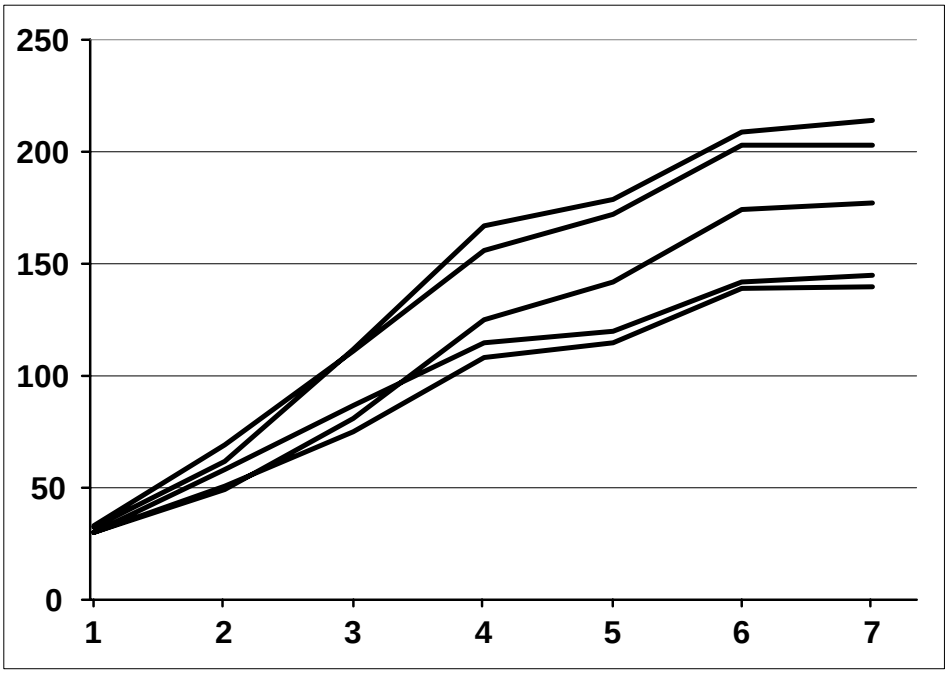
The botanist's objective was to fit a logistic growth model.

The usual Logistic can be expressed as

$$Y_{ij} = \frac{\beta_1}{1+\beta_2 e^{\beta_3 X_i}} + \varepsilon_{ij}$$

Is modified to

$$Y_{ij} = \frac{\beta_1 + \mu_1}{1+(\beta_2+\mu_2) e^{\beta_3 X_i}} + \varepsilon_{ij}$$



```
1622       title2 'Logistic model fit with covariance';
1623       %nlinmix(data=tree,
1624          response=y,
1625          subject=tree,
1626          model=%str(
1627             num = b1+u1;
1628             e = exp(b3*x);
1629             den = 1 + (b2+u2)*e;
1630             pred = num/den;
1631          ),
1632          derivs=%str(
1633             d_b1 = 1/den;
1634             d_b2 = -num/den/den*e;
1635             d_b3 = -num/den/den*b2*x*e;
1636             d_u1 = d_b1;
1637             d_u2 = d_b2;
1638          ),
1639          parms=%str(b1=150 b2=10 b3=-.001),
1640          random=u1 u2,
1641          type=un,
1642          expand=zero
1643       )
                        The NLINMIX Macro
           Data Set                     : WORK.TREE
           Response                     : Y
           Subject                      : TREE
           Fixed Effects Parameters     : B1 B2 B3
           Random Effects Parameters    : U1 U2
           Covariance Type              : UN
           Expansion Point              : ZERO
           Optimization Method          : REML

                         Dimensions
                Observations                : 35
                Subjects                    : 5
                Maximum Obs per Subject     : 7
                Fixed Effects Parameters    : 3
                Random Effects per Subject  : 2

Calling PROC NLIN to initialize.
Iteratively calling PROC MIXED.
   PROC MIXED call 0
iteration = 0
convergence criterion = .
B1=192.68769779 B2=7.8565547794 B3=-0.002828581 COVP1=1578.8110866 COVP2=32.9435116
COVP3=1.2060311175 COVP4=57.969882337
   PROC MIXED call 1
iteration = 1
convergence criterion = 3.4794365E-7
B1=192.68759742 B2=7.8565632683 B3=-0.002828585 COVP1=1578.8105373 COVP2=32.943500137
COVP3=1.2060306978 COVP4=57.969862167
   PROC MIXED call 2
iteration = 2
convergence criterion = 1.7989161E-8
B1=192.68759222 B2=7.8565637054 B3=-0.002828585 COVP1=1578.8105089 COVP2=32.943499545
COVP3=1.2060306761 COVP4=57.969861124
   PROC MIXED call 3
iteration = 3
convergence criterion = 9.293481E-10
B1=192.68759196 B2=7.856563728 B3=-0.002828585 COVP1=1578.8105074 COVP2=32.943499514
COVP3=1.206030675 COVP4=57.96986107
NLINMIX convergence criteria met.
NOTE: PROCEDURE DATASETS used:
      real time           0.00 seconds
NOTE: Numeric values have been converted to character values at the places given by:
      (Line):(Column).
      1643:50    1643:133
NOTE: There were 35 observations read from the data set WORK._NLINMIX.
```

```
NOTE: The data set WORK._NLINMIX has 35 observations and 22 variables.
NOTE: DATA statement used:
      real time           0.00 seconds
   PROC MIXED call 4
NOTE: Convergence criteria met.
NOTE: The data set WORK._FIT has 4 observations and 2 variables.
NOTE: The data set WORK._SOLNR has 10 observations and 7 variables.
NOTE: The data set WORK._COV has 4 observations and 3 variables.
NOTE: The data set WORK._SOLN has 3 observations and 9 variables.
NOTE: The PROCEDURE MIXED printed page 1.
NOTE: PROCEDURE MIXED used:
      real time           0.22 seconds
NOTE: Numeric values have been converted to character values at the places given by:
      (Line):(Column).
      1643:51
NOTE: There were 4 observations read from the data set WORK._FIT.
NOTE: DATA statement used:
      real time           0.00 seconds
1644
1645       run;
1646


NLINMix - from SAS System for Mixed Models, 1996, Littell, et al.
1
Logistic model fit with covariance

The Mixed Procedure
                  Model Information
Data Set                      WORK._NLINMIX
Dependent Variable            _resid
Covariance Structure          Unstructured
Subject Effect                _subject
Estimation Method             REML
Residual Variance Method      Profile
Fixed Effects SE Method       Model-Based
Degrees of Freedom Method     Containment

Class Level Information
Class       Levels    Values
_subject         5    1 2 3 4 5

            Dimensions
Covariance Parameters             4
Columns in X                      3
Columns in Z Per Subject          2
Subjects                          5
Max Obs Per Subject               7
Observations Used                35
Observations Not Used             0
Total Observations               35

Parameter Search
   CovP1       CovP2       CovP3       CovP4    Variance            Res Log Like    -2 Res Log Like
 1578.81     32.9435      1.2060     57.9699     57.9699               -134.2208           268.4416

Iteration History
Iteration    Evaluations    -2 Res Log Like       Criterion
        1              1       268.44156466      0.00000000
Convergence criteria met.

Covariance Parameter Estimates
Cov Parm     Subject     Estimate
UN(1,1)      _subject     1578.81
UN(2,1)      _subject     32.9435
UN(2,2)      _subject      1.2060
Residual                  57.9699
```

```
Fit Statistics
-2 Res Log Likelihood           268.4
AIC (smaller is better)         276.4
AICC (smaller is better)        277.9
BIC (smaller is better)         274.9

PARMS Model Likelihood Ratio Test
    DF    Chi-Square      Pr > ChiSq
     3          0.00          1.0000

Solution for Fixed Effects
                     Standard
Effect    Estimate       Error      DF    t Value    Pr > |t|     Alpha       Lower       Upper
D_B1        192.69     18.9540      24      10.17      <.0001      0.05      153.57      231.81
D_B2        7.8566      0.9320      24       8.43      <.0001      0.05      5.9330      9.7802
D_B3      -0.00283    0.000212      24     -13.32      <.0001      0.05    -0.00327    -0.00239

                        Solution for Random Effects

                                      Std Err
Effect    _subject    Estimate         Pred      DF    t Value    Pr > |t|

D_U1      1           -35.9050      18.5027      24      -1.94      0.0642
D_U2      1            -1.1077       0.7636      24      -1.45      0.1599
D_U1      2            33.7187      18.5027      24       1.82      0.0809
D_U2      2             0.3642       0.7636      24       0.48      0.6377
D_U1      3           -41.5221      18.5027      24      -2.24      0.0343
D_U2      3            -0.7687       0.7636      24      -1.01      0.3242
D_U1      4            45.0585      18.5027      24       2.44      0.0227
D_U2      4             0.8581       0.7636      24       1.12      0.2723
D_U1      5            -1.3500      18.5027      24      -0.07      0.9424
D_U2      5             0.6541       0.7636      24       0.86      0.4002



1647       title2 'Logistic model fit without covariance';
1648
1649
1650       %nlinmix(data=tree,
1651          response=y,
1652          subject=tree,
1653          model=%str(
1654             num = b1+u1;
1655             e = exp(b3*x);
1656             den = 1 + (b2+u2)*e;
1657             pred = num/den;
1658          ),
1659          derivs=%str(
1660             d_b1 = 1/den;
1661             d_b2 = -num/den/den*e;
1662             d_b3 = -num/den/den*b2*x*e;
1663             d_u1 = d_b1;
1664             d_u2 = d_b2;
1665          ),
1666          parms=%str(b1=150 b2=10 b3=-.001),
1667          random=u1 u2,
1668          expand=zero
1669       )

                        The NLINMIX Macro
             Data Set                     : WORK.TREE
             Response                     : Y
             Subject                      : TREE
             Fixed Effects Parameters     : B1 B2 B3
             Random Effects Parameters    : U1 U2
             Covariance Type              : VC
             Expansion Point              : ZERO
             Optimization Method          : REML
```

```
                    Dimensions
               Observations               : 35
               Subjects                   : 5
               Maximum Obs per Subject    : 7
               Fixed Effects Parameters   : 3
               Random Effects per Subject : 2

Calling PROC NLIN to initialize.
Iteratively calling PROC MIXED.
   PROC MIXED call 0
iteration = 0
convergence criterion = .
B1=192.68769779 B2=7.8565547794 B3=-0.002828581 COVP1=1377.1254461 COVP2=0.8774115945
COVP3=59.355858459
   PROC MIXED call 1
iteration = 1
convergence criterion = 0.0002309014
B1=192.68759742 B2=7.8565632683 B3=-0.002828585 COVP1=1377.1312916 COVP2=0.8776424958
COVP3=59.354818653
   PROC MIXED call 2
iteration = 2
convergence criterion = 1.2367313E-8
B1=192.68759222 B2=7.8565637054 B3=-0.002828585 COVP1=1377.1312745 COVP2=0.877642485
COVP3=59.354817919
   PROC MIXED call 3
iteration = 3
convergence criterion = 6.389805E-10
B1=192.68759196 B2=7.856563728 B3=-0.002828585 COVP1=1377.1312737 COVP2=0.8776424844
COVP3=59.354817881
NLINMIX convergence criteria met.
NOTE: PROCEDURE DATASETS used:
      real time           0.00 seconds
NOTE: Numeric values have been converted to character values at the places given by:
      (Line):(Column).
      1669:50    1669:133
NOTE: There were 35 observations read from the data set WORK._NLINMIX.
NOTE: The data set WORK._NLINMIX has 35 observations and 22 variables.
NOTE: DATA statement used:
      real time           0.00 seconds
   PROC MIXED call 4
NOTE: Convergence criteria met.
NOTE: The data set WORK._FIT has 4 observations and 2 variables.
NOTE: The data set WORK._SOLNR has 10 observations and 7 variables.
NOTE: The data set WORK._COV has 3 observations and 3 variables.
NOTE: The data set WORK._SOLN has 3 observations and 9 variables.
NOTE: The PROCEDURE MIXED printed page 2.
NOTE: PROCEDURE MIXED used:
      real time           0.11 seconds
NOTE: Numeric values have been converted to character values at the places given by:
      (Line):(Column).
      1669:51
NOTE: There were 4 observations read from the data set WORK._FIT.
NOTE: DATA statement used:
      real time           0.00 seconds

NLINMix - from SAS System for Mixed Models, 1996, Littell, et al.
Logistic model fit without covariance

The Mixed Procedure
                  Model Information
Data Set                      WORK._NLINMIX
Dependent Variable            _resid
Covariance Structure          Variance Components
Subject Effect                _subject
Estimation Method             REML
Residual Variance Method      Profile
Fixed Effects SE Method       Model-Based
Degrees of Freedom Method     Containment
```

Class Level Information

Class	Levels	Values
_subject	5	1 2 3 4 5

Dimensions

Covariance Parameters	3
Columns in X	3
Columns in Z Per Subject	2
Subjects	5
Max Obs Per Subject	7
Observations Used	35
Observations Not Used	0
Total Observations	35

Parameter Search

CovP1	CovP2	CovP3	Variance	Res Log Like	-2 Res Log Like
1377.13	0.8776	59.3548	59.3548	-134.9425	269.8851

Iteration History

Iteration	Evaluations	-2 Res Log Like	Criterion
1	1	269.88509739	0.00000000

Convergence criteria met.

Covariance Parameter Estimates

Cov Parm	Subject	Estimate
D_U1	_subject	1377.13
D_U2	_subject	0.8776
Residual		59.3548

Fit Statistics

-2 Res Log Likelihood	269.9
AIC (smaller is better)	275.9
AICC (smaller is better)	276.7
BIC (smaller is better)	274.7

PARMS Model Likelihood Ratio Test

DF	Chi-Square	Pr > ChiSq
2	0.00	1.0000

Solution for Fixed Effects

Effect	Estimate	Standard Error	DF	t Value	Pr > |t|	Alpha	Lower	Upper
D_B1	192.69	17.8873	24	10.77	<.0001	0.05	155.77	229.61
D_B2	7.8566	0.9044	24	8.69	<.0001	0.05	5.9899	9.7232
D_B3	-0.00283	0.000215	24	-13.16	<.0001	0.05	-0.00327	-0.00239

Solution for Random Effects

Effect	_subject	Estimate	Std Err Pred	DF	t Value	Pr > |t|
D_U1	1	-33.7537	17.3736	24	-1.94	0.0639
D_U2	1	-0.7596	0.7648	24	-0.99	0.3305
D_U1	2	31.2403	17.3736	24	1.80	0.0847
D_U2	2	-0.1495	0.7648	24	-0.20	0.8467
D_U1	3	-38.6871	17.3736	24	-2.23	0.0356
D_U2	3	-0.2246	0.7648	24	-0.29	0.7716
D_U1	4	41.9983	17.3736	24	2.42	0.0236
D_U2	4	0.2743	0.7648	24	0.36	0.7230
D_U1	5	-0.7978	17.3736	24	-0.05	0.9638
D_U2	5	0.8594	0.7648	24	1.12	0.2723

Full model Fit Statistics : -2 Res Log Likelihood 268.4
Reduced model Fit Statistics : -2 Res Log Likelihood 269.9
Difference 269.9 – 268.4 = 1.5 with 1 d.f. , $P>\chi^2 = 0.220671492$