```
1          ********************************************************;
2          *** Logistic Regression - Disease outbreak example     ***;
3          *** NKNW table 14.3  (Appendix C3)                     ***;
4          *** Study of a disease outbreak from a mosquito born   ***;
5          *** disease within two sectors of a city.              ***;
6          ********************************************************;
7
8          dm'log;clear;output;clear';
9          options nodate nocenter nonumber ps=512 ls=132 nolabel;
10         ODS HTML style=minimal rs=none body='C:\Geaghan\Current\EXST7034\Fall2005\SAS\DiseaseOutbreak01.html' ;
NOTE: Writing HTML Body file: C:\Geaghan\Current\EXST7034\Fall2005\SAS\DiseaseOutbreak01.html
11
12         TITLE1 'Logistic Regression - NKNW Example 14.3';
13         data Disease; infile cards missover;
14            input case Age Status1 Status2 sector Disease;
15            * Status classes are upper (0, 0), Middle (1, 0) and Lower (0, 1);
16            status = 'Upper ';
17            if status1 eq 1 then status = 'Middle';
18            if status2 eq 1 then status = 'Lower';
19            label case = 'case number'
20                  age  = 'Patients age'
21                  status  = 'Socioeconomic status upper, middle and lower'
22                  disease = 'Disease present = 1';
23         Cards;
NOTE: The data set WORK.DISEASE has 98 observations and 7 variables.
NOTE: DATA statement used (Total process time):
      real time           0.02 seconds
      cpu time            0.02 seconds
122        ;
123
124        proc logistic data=Disease DESCENDING alpha=0.05;
125           TITLE2 'Logistic regression on Disease data (with Status1 and Status2)';
126           model Disease = Age Status1 Status2 Sector;
127        run;
NOTE: PROC LOGISTIC is modeling the probability that Disease=1.
NOTE: Convergence criterion (GCONV=1E-8) satisfied.
NOTE: There were 98 observations read from the data set WORK.DISEASE.
NOTE: The PROCEDURE LOGISTIC printed page 1.
NOTE: PROCEDURE LOGISTIC used (Total process time):
      real time           0.25 seconds
      cpu time            0.04 seconds
128
129        proc logistic data=Disease DESCENDING alpha=0.05;
130           class status;
131           TITLE2 'Logistic regression on Disease data (with CLASS Status - default)';
132           model Disease = Age Status Sector;
133        run;
NOTE: PROC LOGISTIC is modeling the probability that Disease=1.
NOTE: Convergence criterion (GCONV=1E-8) satisfied.
NOTE: There were 98 observations read from the data set WORK.DISEASE.
NOTE: The PROCEDURE LOGISTIC printed page 2.
NOTE: PROCEDURE LOGISTIC used (Total process time):
      real time           0.19 seconds
      cpu time            0.03 seconds
134
135        proc logistic data=Disease DESCENDING alpha=0.05;
136           class status / param = glm;
137           TITLE2 'Logistic regression on Disease data (with CLASS Status - GLM)';
138           model Disease = Age Status Sector / lackfit RSQ iplots;
139           output out=next1 PREDICTED=yhat Lower=lcl95 Upper=ucl95 dfbetas=_ALL_
140                  resdev=resdev difdev=difdev;
141        run;
NOTE: PROC LOGISTIC is modeling the probability that Disease=1.
NOTE: Convergence criterion (GCONV=1E-8) satisfied.
NOTE: There were 98 observations read from the data set WORK.DISEASE.
NOTE: The data set WORK.NEXT1 has 98 observations and 19 variables.
NOTE: The PROCEDURE LOGISTIC printed pages 3-4.
```

```
NOTE: PROCEDURE LOGISTIC used (Total process time):
      real time           0.23 seconds
      cpu time            0.10 seconds
```

The three PROC LOGISTIC analyses differ in only one regard, and all results are the same except the parameter estimates. There are 3 socio-economic levels and many different ways of setting up the dummy variables. Several are explored below. Make sure you know what you are estimating.

- The first version uses the status dummy variables provided with the data set where two variables status1 and status2 have the following values for the 3 socio-economic levels: Upper (0, 0), Middle (1, 0) and Lower (0, 1).
- I created a class variable STATUS with the three levels coded as "Lower, Middle, Upper". In the second PROC LOGISTIC the STATUS variable was placed in the class statement.
- In the third PROC LOGISTIC, for which a full analysis is provided, the option "/ param = glm;" was requested on the CLASS statement.

Logistic Regression - NKNW Example 14.3
Logistic regression on Disease data (with Status1 and Status2)

The LOGISTIC Procedure

Analysis of Maximum Likelihood Estimates

Parameter	DF	Estimate	Standard Error	Wald Chi-Square	Pr > ChiSq
Intercept	1	-2.3127	0.6426	12.9545	0.0003
Age	1	0.0297	0.0135	4.8535	0.0276
Status1	1	0.4088	0.5990	0.4657	0.4950
Status2	1	-0.3051	0.6041	0.2551	0.6135
sector	1	1.5746	0.5016	9.8543	0.0017

Odds Ratio Estimates

Effect	Point Estimate	95% Wald Confidence Limits	
Age	1.030	1.003	1.058
Status1	1.505	0.465	4.868
Status2	0.737	0.226	2.408
sector	4.829	1.807	12.907

Logistic Regression - NKNW Example 14.3
Logistic regression on Disease data (with CLASS Status - default)

The LOGISTIC Procedure

Class Level Information

Class	Value	Design Variables	
status	Lower	1	0
	Middle	0	1
	Upper	-1	-1

Note the coding of the dummy variables, contrasting Lower and Middle to Upper.

Analysis of Maximum Likelihood Estimates

Parameter		DF	Estimate	Standard Error	Wald Chi-Square	Pr > ChiSq
Intercept		1	-2.2782	0.5195	19.2314	<.0001
Age		1	0.0297	0.0135	4.8535	0.0276
status	Lower	1	-0.3397	0.3690	0.8471	0.3574
status	Middle	1	0.3742	0.3662	1.0439	0.3069
sector		1	1.5746	0.5016	9.8543	0.0017

Odds Ratio Estimates

Effect	Point Estimate	95% Wald Confidence Limits	
Age	1.030	1.003	1.058
status Lower vs Upper	0.737	0.226	2.408
status Middle vs Upper	1.505	0.465	4.868
sector	4.829	1.807	12.907

Logistic Regression - NKNW Example 14.3
Logistic regression on Disease data (with CLASS Status - GLM)

The LOGISTIC Procedure

Model Information

Data Set	WORK.DISEASE
Response Variable	Disease
Number of Response Levels	2
Model	binary logit
Optimization Technique	Fisher's scoring

Number of Observations Read	98
Number of Observations Used	98

Response Profile

Ordered Value	Disease	Total Frequency
1	1	31
2	0	67

Probability modeled is Disease=1.

Class Level Information

Class	Value	Design Variables		
status	Lower	1	0	0
	Middle	0	1	0
	Upper	0	0	1

Note the GLM-like coding of the dummy variables.

Model Convergence Status
Convergence criterion (GCONV=1E-8) satisfied.

Model Fit Statistics

Criterion	Intercept Only	Intercept and Covariates
AIC	124.318	111.054
SC	126.903	123.979
-2 Log L	122.318	101.054

R-Square 0.1950 Max-rescaled R-Square 0.2736

Model fit statistics

1) Akaike Information Criterion $\text{AIC} = -2\log(L) + 2p$

where Log(L) is the log likelihood and p is the number of parameters

2) Schwarz Criterion $\text{SC} = -2\log(L) + p\log(\sum_j f_j)$

3) –2log L $-2\sum_{i-1}^{n}\left[Y_i \log_e(\hat{\pi}_i) + (1-Y_i)\log_e(1-\hat{\pi}_i)\right]$

This is analogous to the SSE in regression and is given in SAS as the "–2 Log L".

Two models (full and reduced) can be compared by calculating the difference in "–2 Log L" for both models. This difference follows a chi square distribution with a d.f. equal to the difference in d.f. for the two models.

4) Generalized R^2 $1-\left(\frac{L(0)}{L(\theta)}\right)^{\frac{2}{n}}$, where L(0) is the intercept only model.

Since this value reaches its maximum of less than 1 for discrete models an adjustment has been proposed. This is called the Max-rescaled Rsquare in SAS. $\frac{R^2}{R^2_{\max}}$

```
Testing Global Null Hypothesis: BETA=0
Test                 Chi-Square       DF     Pr > ChiSq
Likelihood Ratio        21.2635        4         0.0003
Score                   20.4067        4         0.0004
Wald                    16.6437        4         0.0023
```

All three tests are joint tests of the regression coefficients (β_i) against 0.

Likelihood ratio test – This test compares the -2 Log L values for the model with an "intercept only model" (see Model fit statistics above). The difference (122.318 –101.054) follows a chi square distribution with degrees of freedom equal to p – 1, the number of parameters not counting the intercept.

This is analogous to the SSE in regression and is given in SAS as the "-2 Log L".

Any two models (full and reduced) can be compared by calculating the difference in "-2 Log L" for both models. This difference follows a chi square distribution with a d.f. equal to the difference in d.f. for the two models. This test is analogous to the General Linear Hypothesis test for linear models.

Score statistics – These are based on vectors of the partial derivatives of the log likelihood (wrt the parameter vector) and the matrix of second partial derivatives. It asymptotically has a chi square distribution.

Wald statistics – are based on large sample statistics and assume asymptotic normality.

Under the usual large sample conditions the logistic regression maximum likelihood estimators are approximately normally distributed, have no bias and have variances and covariances that are functions of the partial second derivative (Hessian) of the log likelihood function.

Type 3 Analysis of Effects

Effect	DF	Wald Chi-Square	Pr > ChiSq
Age	1	4.8535	0.0276
status	2	1.2053	0.5474
sector	1	9.8543	0.0017

Analysis of Maximum Likelihood Estimates

Parameter		DF	Estimate	Standard Error	Wald Chi-Square	Pr > ChiSq
Intercept		1	-2.3127	0.6426	12.9545	0.0003
Age		1	0.0297	0.0135	4.8535	0.0276
status	Lower	1	-0.3051	0.6041	0.2551	0.6135
status	Middle	1	0.4088	0.5990	0.4657	0.4950
status	Upper	0	0	.	.	.
sector		1	1.5746	0.5016	9.8543	0.0017

Wald : used to test individual parameter estimates and to place confidence intervals. It is based on a large sample assumption of asymptotic normality.

Chi-square Test $\beta_i^2 / Var(\beta_i) = [\beta_i / Stderr(\beta_i)]^2$

Confidence interval $P\left(e^{(\hat{\beta}_i - 1.96 S_{\hat{\beta}_i})} \le \beta_i \le e^{(\hat{\beta}_i - 1.96 S_{\hat{\beta}_i})}\right) = 0.95$

Odds Ratio Estimates

Effect	Point Estimate	95% Wald Confidence Limits	
Age	1.030	1.003	1.058
status Lower vs Upper	0.737	0.226	2.408
status Middle vs Upper	1.505	0.465	4.868
sector	4.829	1.807	12.907

Association of Predicted Probabilities and Observed Responses

Percent Concordant	77.5	Somers' D	0.554
Percent Discordant	22.1	Gamma	0.556
Percent Tied	0.3	Tau-a	0.242
Pairs	2077	c	0.777

Association of Predicted Probabilities and Observed Responses

Observations with different responses are paired and compared. If the one with the lower observed response has a lower predicted value it is said to be concordant. Otherwise they are discordant or tied. SAS reports concordant, discordant, ties and the number of pairs examined.

A number of other statistics are based on the same information of the number concordant (n_c) and the number discordant (n_d). Where "t" is the total number of pairs with different responses and N is the sum of observation frequencies in the data then the following statistics can be derived. Note that ties are given by $t-n_c-n_d$. The statistic "c" is equal to $(n_c + 0.5(t-n_c-n_d))/t$. Somers' D is equal to $(n_c-n_d)/t$. The Goodman-Kruskal Gamma is $(n_c-n_d)/(n_c+n_d)$ and Kendall's Tau-a is $(n_c-n_d)/(0.5N(N-1)/$

Partition for the Hosmer and Lemeshow Test

		Disease = 1		Disease = 0	
Group	Total	Observed	Expected	Observed	Expected
1	10	0	0.79	10	9.21
2	10	1	1.02	9	8.98
3	11	2	1.51	9	9.49
4	10	1	1.78	9	8.22
5	10	3	2.34	7	7.66
6	10	4	3.09	6	6.91
7	10	7	3.91	3	6.09
8	11	3	5.51	8	5.49
9	10	5	6.32	5	3.68
10	6	5	4.75	1	1.25

Hosmer and Lemeshow Goodness-of-Fit Test

Chi-Square	DF	Pr > ChiSq
9.1871	8	0.3268

Goodness-of-fit

Most Goodness-of-fit tests (Pearson and deviance) require replication in subpopulations. This is often a problem with continuous variables in the model. The Hosmer-Lemeshow Goodness-of-Fit Test can be used for sparser data. This test is only available for binary models.

In this approach the data are sorted on the basis of their response probability (default), and divided into approximately 10 groups (minimum = 3). See SAS help for details on the grouping.

Once in groups a Chi square statistic is calculated. For each group we have S_i as the observed number of "successes" in the group, n_i as the total number of observations in the group and $\bar{\pi}_i$ as the mean predicted probability in each group (from the model). For the "g" groups the Chi square statistic is then calculated as:

$$\chi^2 = \sum_{i=1}^{g} \frac{(S_i - n_i\bar{\pi}_i)^2}{n_i\bar{\pi}_i(1-\bar{\pi}_i)}$$

The usual interpretations apply to this lack of fit test. Small values of P would indicate an inadequate model.

Deviance – The deviance in logistic regression can be calculated as

$$DEV(X_0, X_1, ..., X_{p-1}) = -2\sum_{i-1}^{n}\left[Y_i \log_e(\hat{\pi}_i) + (1-Y_i)\log_e(1-\hat{\pi}_i)\right]$$

Partial deviances can be calculated for reduced models and the difference in the two models (with n−p and n−p+*diff* d.f.) should follow a Chi square distribution with *diff* d.f. This test of partial deviances is also called the likelihood ratio test.

Deviance residuals – Residual analysis is not as simple with Logistic Regression as it was with Linear Regression. Since the dependent variable is 0 or 1 they are not normally distributed and in fact the distribution is not known. As a result residual analysis in Logistic Regression is done on "Deviance" residuals. These are calculated as: $dev_i = \pm\sqrt{\{-2[Y_i \log_e(\hat{\pi}_i) + (1-Y_i)\log_e(1-\hat{\pi}_i)]\}}$

where the sign is + if $Y_i \geq \hat{\pi}_i$ and the sign is - if $Y_i < \hat{\pi}_i$.

Model deviance – the SS of these residuals will sum to the model deviance.

Pearson residuals – Given the residual $e_i = Y_i - \hat{\pi}_i$, the predicted probabilities $\hat{\pi}_i = Y_i - e_i$ and r_i = the number of events in a given observation with n_i trials, then the Pearson residual is given as:

$$\frac{r_i - n_i\hat{\pi}_i}{\sqrt{n_i\hat{\pi}_i(1-\hat{\pi}_i)}}$$

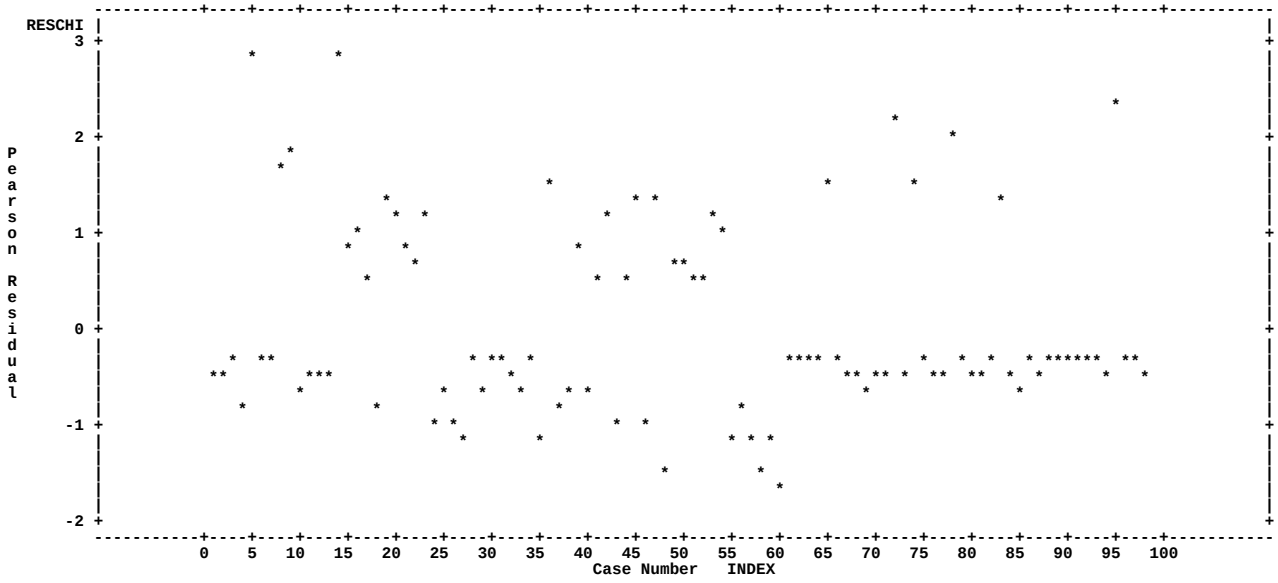
Confidence interval displacement diagnostics – SAS provides two measures of confidence interval displacement as influence diagnostics, C for individual observations and CBAR for overall change in the parameter estimates when individual observations are removed.

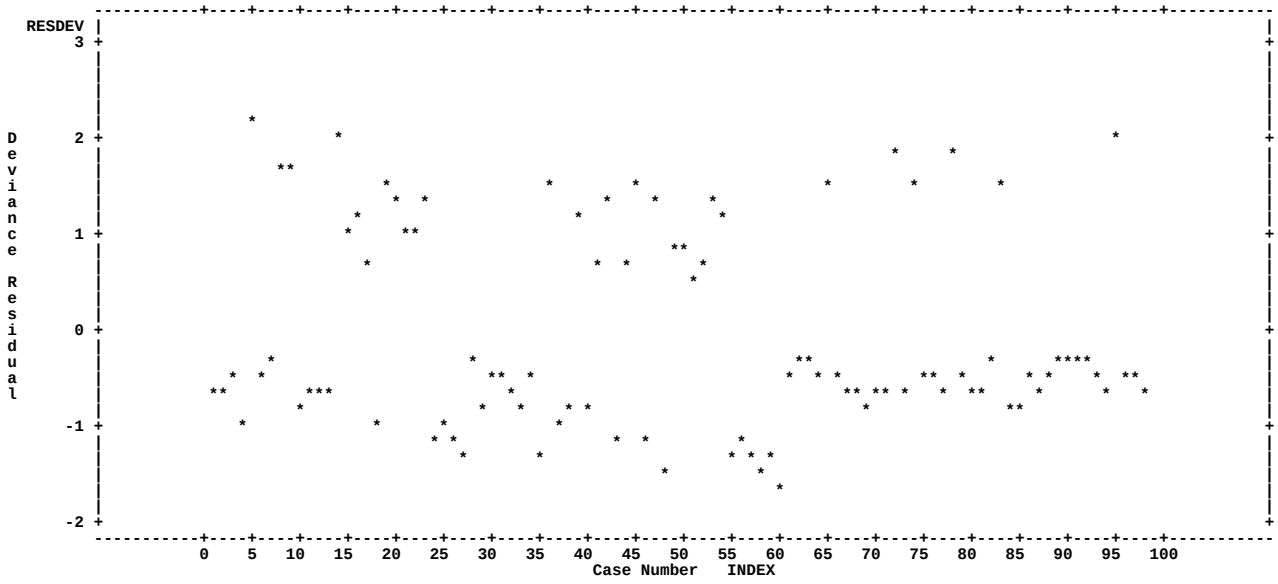
```
143        proc sort data=next1 nodupkey; by Age status; run;
NOTE: There were 98 observations read from the data set WORK.NEXT1.
NOTE: 25 observations with duplicate key values were deleted.
NOTE: The data set WORK.NEXT1 has 73 observations and 19 variables.
NOTE: PROCEDURE SORT used (Total process time):
      real time           0.01 seconds
      cpu time            0.02 seconds
144        proc print data=next1;
145             TITLE2 'Listing of one kept value for each by group from Logistic Reg';
146        run;
NOTE: There were 73 observations read from the data set WORK.NEXT1.
NOTE: The PROCEDURE PRINT printed page 5.
NOTE: PROCEDURE PRINT used (Total process time):
      real time           0.11 seconds
      cpu time            0.08 seconds
147
148        options ps=56 ls=111;
149        proc sort data=Disease; by Age; run;
NOTE: There were 98 observations read from the data set WORK.DISEASE.
NOTE: The data set WORK.DISEASE has 98 observations and 7 variables.
NOTE: PROCEDURE SORT used (Total process time):
      real time           0.01 seconds
      cpu time            0.02 seconds
150        proc sort data=next1; by Age; run;
NOTE: Input data set is already sorted, no sorting done.
NOTE: PROCEDURE SORT used (Total process time):
      real time           0.00 seconds
      cpu time            0.01 seconds
151        proc means data=Disease noprint; by Age; var Disease;
152             output out=next3 n=n mean=mean var=var; run;
NOTE: There were 98 observations read from the data set WORK.DISEASE.
NOTE: The data set WORK.NEXT3 has 50 observations and 6 variables.
NOTE: PROCEDURE MEANS used (Total process time):
      real time           0.01 seconds
      cpu time            0.02 seconds
153
154        data three; set next1 next3; run;
NOTE: There were 73 observations read from the data set WORK.NEXT1.
NOTE: There were 50 observations read from the data set WORK.NEXT3.
NOTE: The data set WORK.THREE has 123 observations and 24 variables.
NOTE: DATA statement used (Total process time):
      real time           0.02 seconds
      cpu time            0.03 seconds
155        proc plot data=three;  plot yhat*Age='x' mean*Age='o' / overlay;
156             TITLE2 'Plot of observed means (o) and predicted values (p)';
157        run;
158
159        ods html close;
NOTE: There were 123 observations read from the data set WORK.THREE.
NOTE: The PROCEDURE PLOT printed page 6.
NOTE: PROCEDURE PLOT used (Total process time):
      real time           0.09 seconds
      cpu time            0.03 seconds
NOTE: SAS Institute Inc., SAS Campus Drive, Cary, NC USA 27513-2414
NOTE: The SAS System used:
      real time           6.59 seconds
      cpu time            0.82 seconds
```

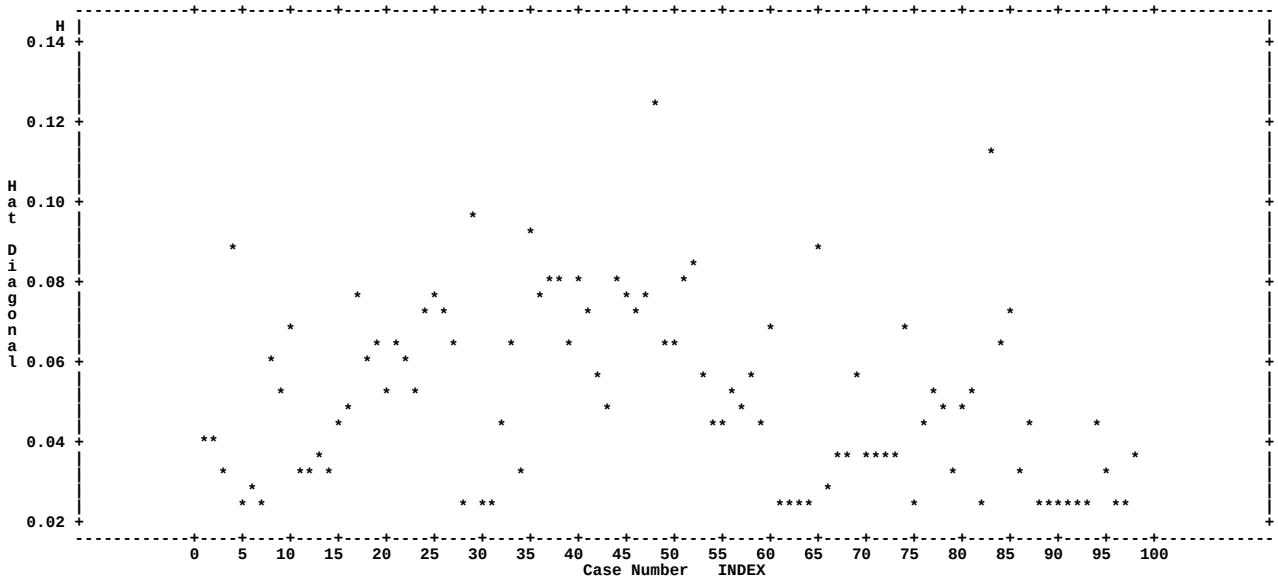
Influence and residual diagnostics are plotted below. There are apparently no generally accepted criteria for evaluation of these statistics in Logistic regression. Your text suggests "subjective visual assessment of an appropriate graphic"

```
Logistic Regression - NKNW Example 14.3
Logistic regression on Disease data

The LOGISTIC Procedure
```

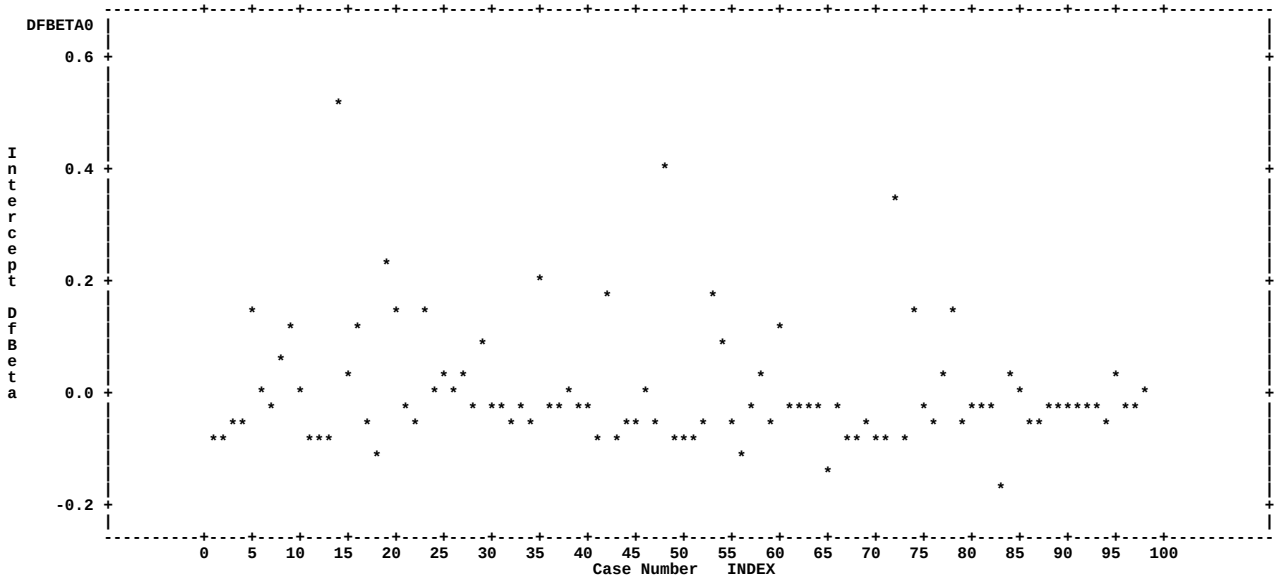


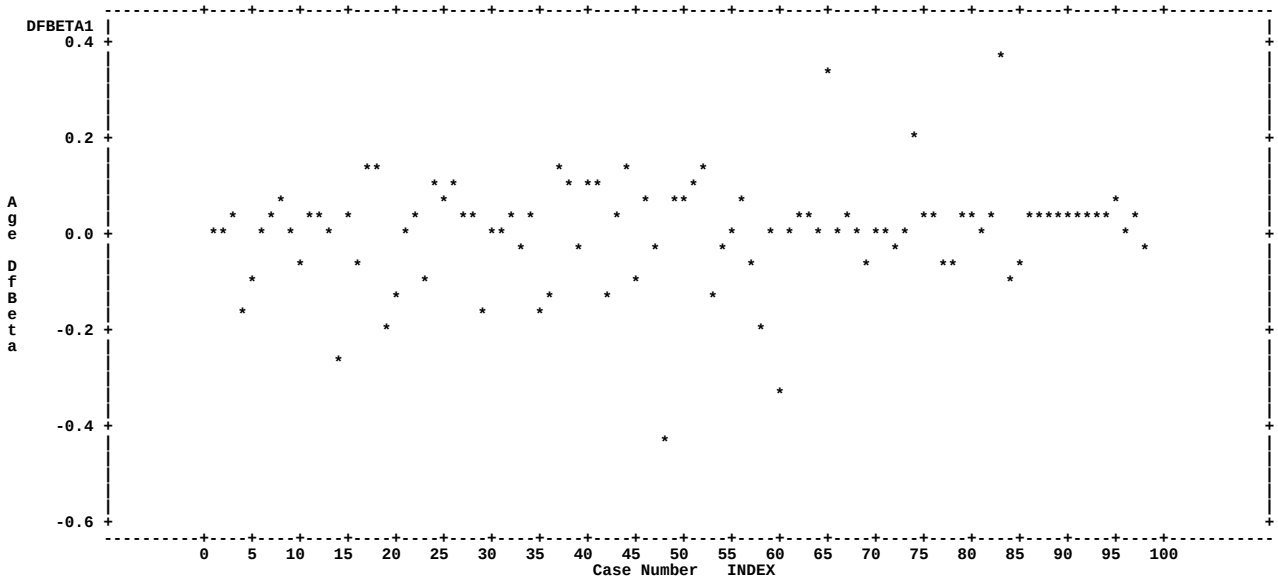


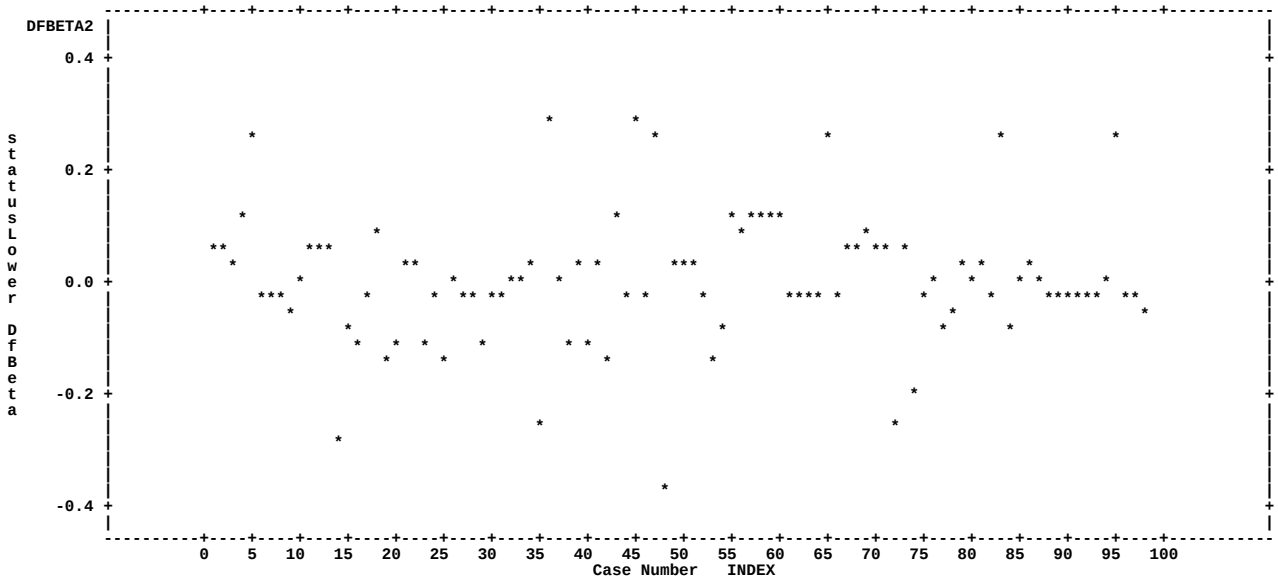


```
Logistic Regression - NKNW Example 14.3
Logistic regression on Disease data

The LOGISTIC Procedure
```

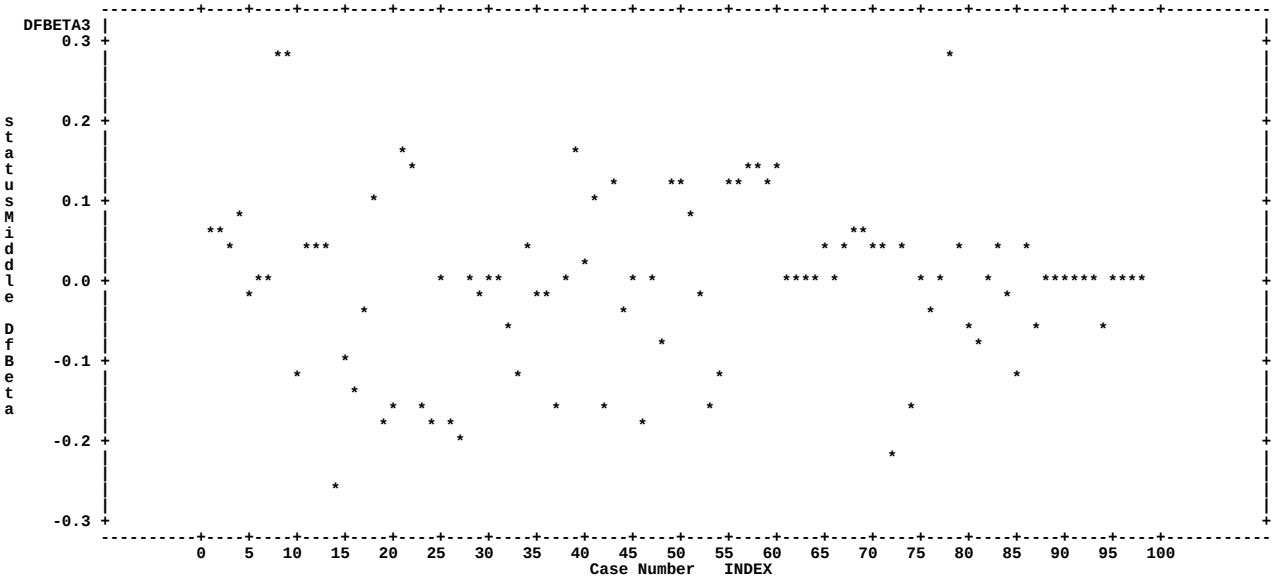


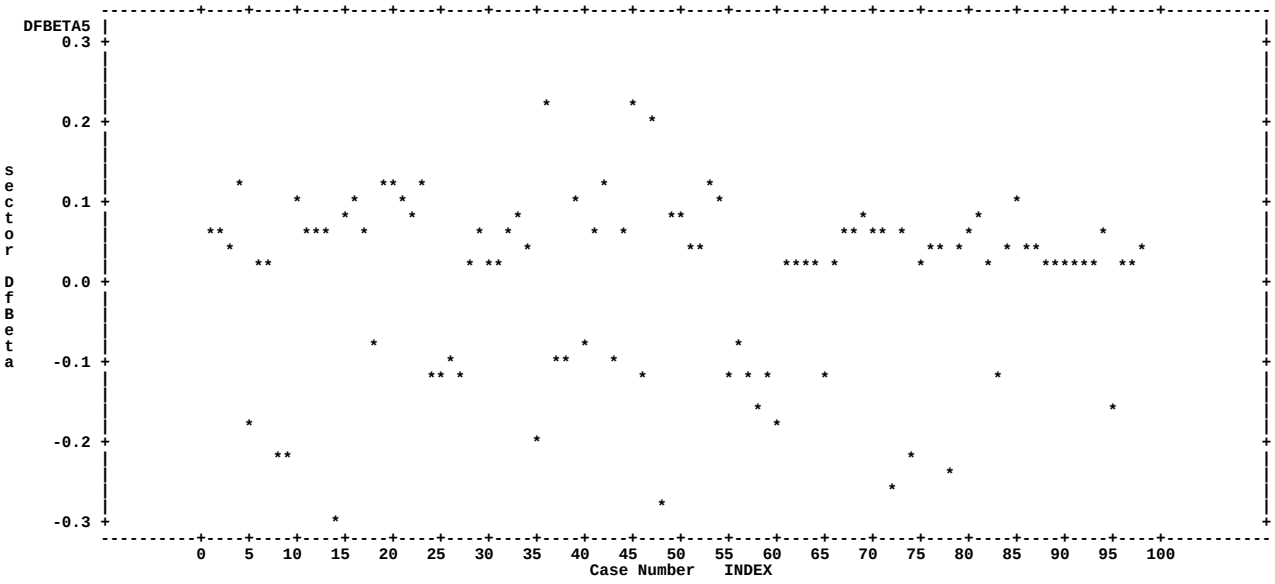


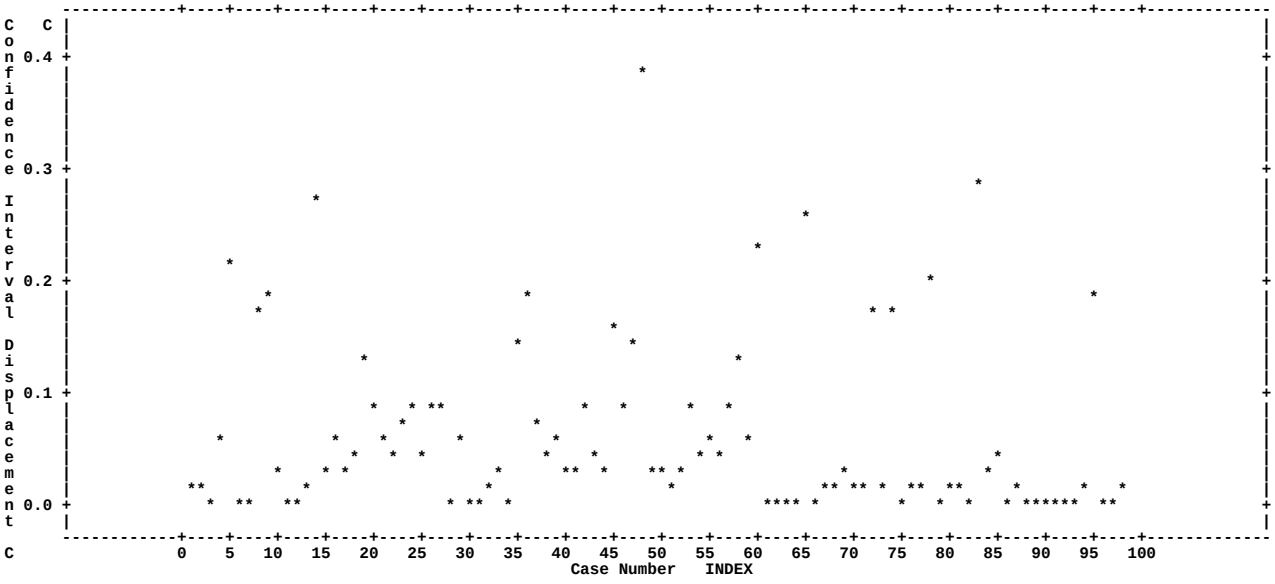


Logistic Regression - NKNW Example 14.3
Logistic regression on Disease data

The LOGISTIC Procedure

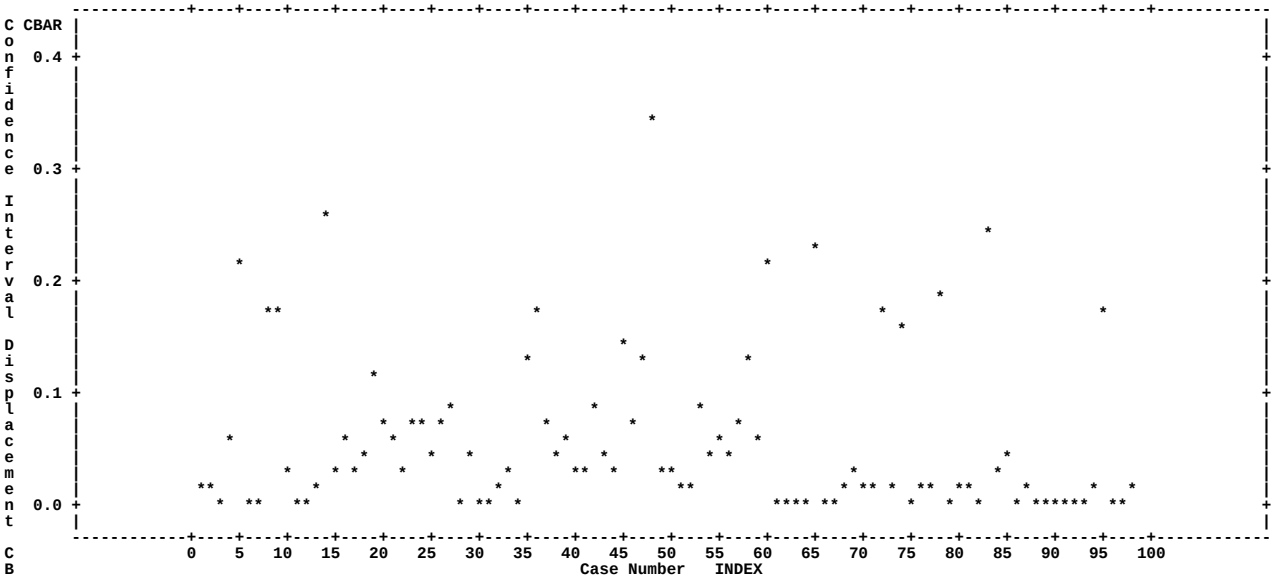


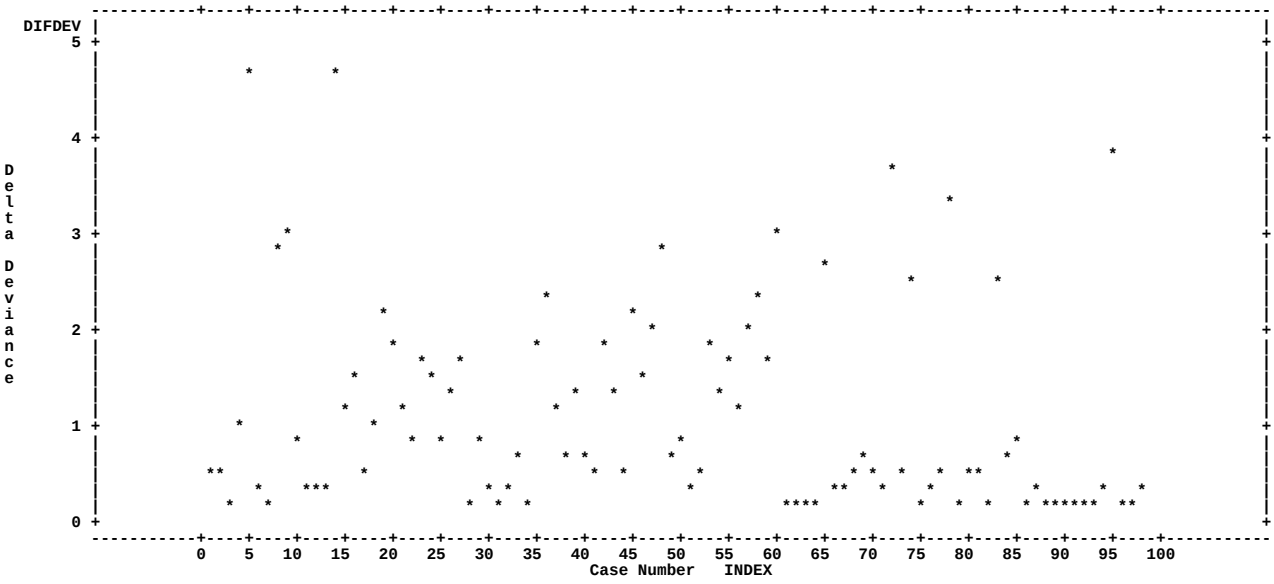


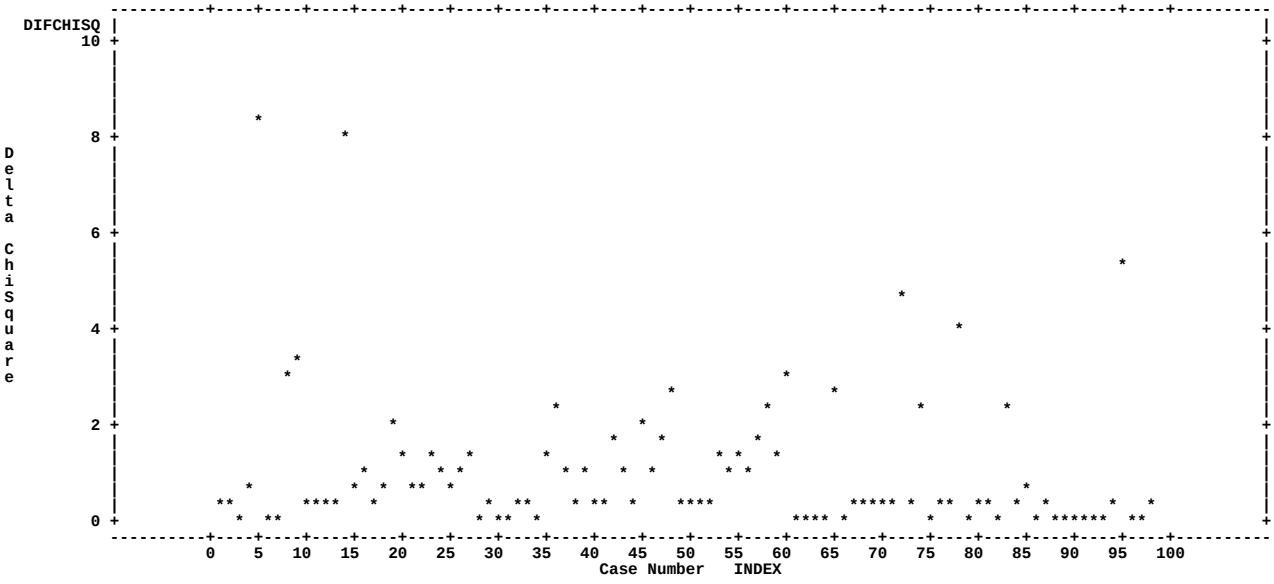


```
Logistic Regression - NKNW Example 14.3
Logistic regression on Disease data

The LOGISTIC Procedure
```







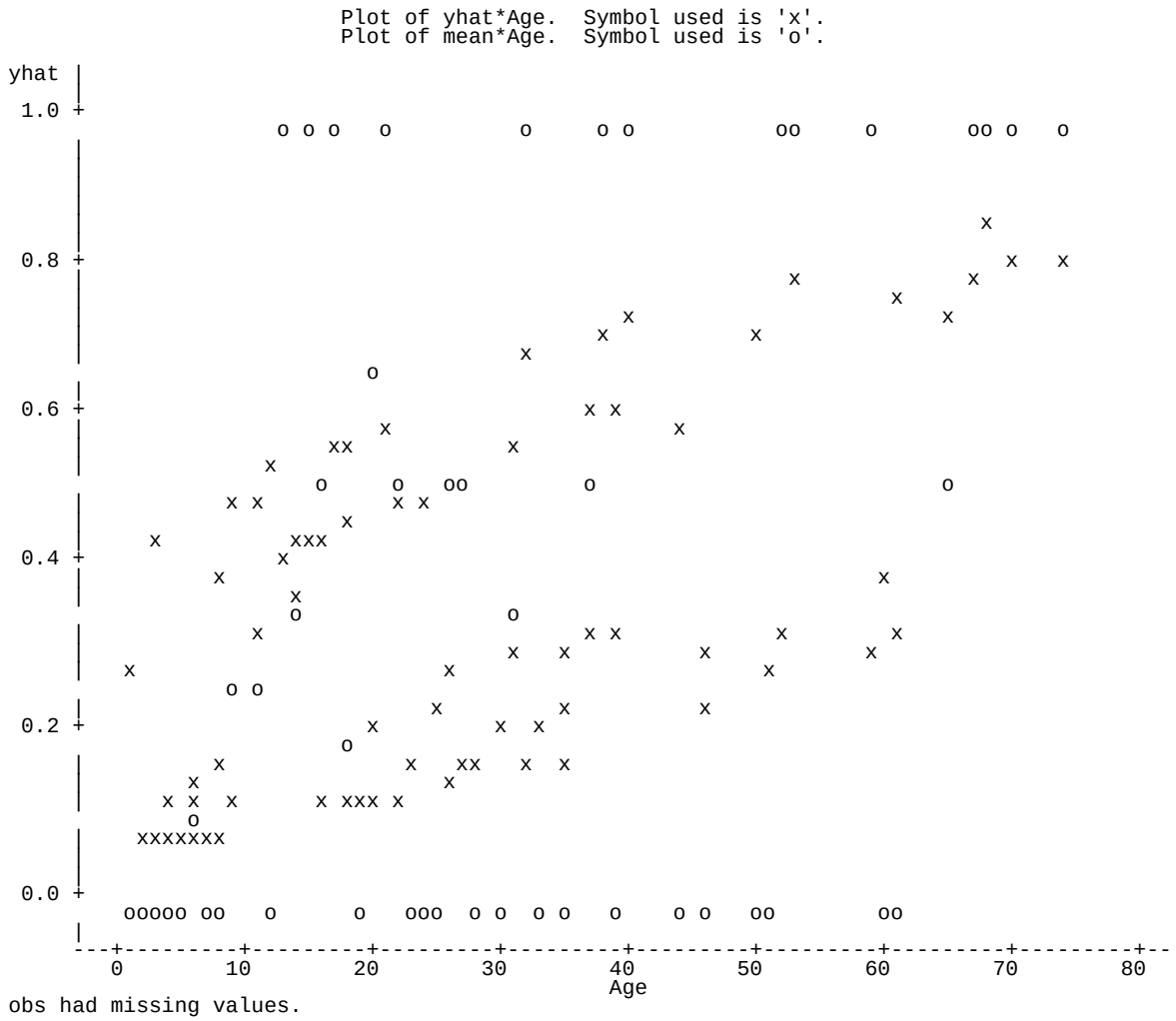
Logistic Regression - NKNW Example 14.3
Listing of one kept value for each by group from Logistic Reg

Obs	case	Age	Status1	Status2	sector	Diseased	status	_LEVEL_	yhat	lcl95	ucl95	resdev	DFBETA_Intercept	DFBETA_Age	DFBETA_statusLower	DFBETA_statusMiddle	DFBETA_statusUpper	DFBETA_sector	difdev
1	40	1	0	1	1	0	Lower	1	0.26630	0.09293	0.56251	-0.78696	-0.01939	0.09579	-0.10891	0.01031	.	-0.08726	0.65146
2	28	2	0	1	0	0	Lower	1	0.07187	0.02346	0.19976	-0.38621	-0.02297	0.02517	-0.02016	0.00406	.	0.01472	0.15105
3	37	3	1	0	1	0	Middle	1	0.44026	0.20277	0.70864	-1.07729	-0.03653	0.13065	-0.00929	-0.15334	.	-0.09777	1.23059
4	79	3	0	0	0	0	Upper	1	0.09766	0.03131	0.26602	-0.45335	-0.06223	0.03867	0.03325	0.03038	.	0.03268	0.20928
5	92	4	0	1	0	0	Lower	1	0.07594	0.02558	0.20460	-0.39743	-0.02289	0.02450	-0.02149	0.00399	.	0.01545	0.15995
6	34	4	0	0	0	0	Upper	1	0.10031	0.03273	0.26867	-0.45980	-0.06299	0.03831	0.03402	0.03101	.	0.03349	0.21527
7	62	5	0	1	0	0	Lower	1	0.07805	0.02670	0.20716	-0.40315	-0.02281	0.02410	-0.02219	0.00395	.	0.01582	0.16460
8	7	6	0	1	0	0	Lower	1	0.08022	0.02785	0.20982	-0.40895	-0.02272	0.02366	-0.02290	0.00390	.	0.01621	0.16937
9	76	6	1	0	0	0	Middle	1	0.15116	0.05238	0.36459	-0.57252	-0.04808	0.04071	0.01268	-0.04816	.	0.04684	0.33631
10	3	6	0	0	0	0	Upper	1	0.10581	0.03573	0.27423	-0.47295	-0.06449	0.03745	0.03561	0.03228	.	0.03517	0.22774
11	91	7	0	1	0	0	Lower	1	0.08244	0.02904	0.21258	-0.41482	-0.02260	0.02318	-0.02364	0.00385	.	0.01660	0.17427
12	93	8	0	1	0	0	Lower	1	0.08472	0.03026	0.21544	-0.42077	-0.02245	0.02265	-0.02440	0.00379	.	0.01700	0.17931
13	87	8	1	0	0	0	Middle	1	0.15896	0.05656	0.37337	-0.58841	-0.04761	0.03832	0.01293	-0.05130	.	0.04903	0.35531
14	18	8	0	0	1	0	Upper	1	0.37751	0.18144	0.62395	-0.97368	-0.12774	0.11676	0.08850	0.10849	.	-0.07308	0.98811
15	26	9	1	0	1	0	Middle	1	0.48460	0.24514	0.73134	-1.15135	-0.01194	0.10015	-0.01394	-0.17329	.	-0.10877	1.40017
16	14	9	0	0	0	1	Upper	1	0.11456	0.04062	0.28333	2.08167	0.51506	-0.27619	-0.29462	-0.26478	.	-0.29239	4.59669
17	36	11	0	1	1	1	Lower	1	0.32827	0.13377	0.60730	1.49259	-0.01603	-0.14208	0.28200	-0.01191	.	0.22377	2.39656
18	24	11	1	0	1	0	Middle	1	0.49946	0.25967	0.73950	-1.17650	-0.00272	0.08844	-0.01563	-0.18022	.	-0.11255	1.46058
19	46	12	1	0	1	0	Middle	1	0.50690	0.26696	0.74370	-1.18915	0.00209	0.08229	-0.01651	-0.18374	.	-0.11446	1.49154
20	42	13	0	0	1	1	Upper	1	0.41304	0.21594	0.64260	1.32982	0.16963	-0.13835	-0.13291	-0.16158	.	0.11495	1.85198
21	25	14	0	1	1	0	Lower	1	0.34824	0.14743	0.62277	-0.92529	0.01835	0.05833	-0.14741	0.00400	.	-0.11664	0.89961
22	53	14	0	0	1	1	Upper	1	0.42027	0.22309	0.64667	1.31670	0.16188	-0.12820	-0.13037	-0.15822	.	0.11381	1.81306
23	20	15	0	0	1	1	Upper	1	0.42754	0.23031	0.65085	1.30362	0.15429	-0.11829	-0.12785	-0.15489	.	0.11266	1.77482
24	31	16	0	1	0	0	Lower	1	0.10509	0.04120	0.24296	-0.47124	-0.02021	0.01648	-0.03134	0.00305	.	0.02053	0.22505
25	23	16	0	0	1	1	Upper	1	0.43483	0.23757	0.65515	1.29058	0.14686	-0.10860	-0.12536	-0.15161	.	0.11150	1.73728
26	39	17	1	0	1	1	Middle	1	0.54397	0.30330	0.76572	1.10351	-0.02357	-0.04073	0.01774	0.16920	.	0.10412	1.27659
27	5	18	0	1	0	1	Lower	1	0.11082	0.04422	0.25134	2.09755	0.15482	-0.11485	0.26748	-0.02231	.	-0.17253	4.60704
28	27	18	1	0	1	0	Middle	1	0.55134	0.31047	0.77032	-1.26609	0.03372	0.04125	-0.02214	-0.20554	.	-0.12619	1.68828
29	56	18	0	0	1	0	Upper	1	0.44951	0.25218	0.66411	-1.09265	-0.10815	0.07342	0.09836	0.11852	.	-0.08914	1.23719
30	96	19	0	1	0	0	Lower	1	0.11378	0.04577	0.25578	-0.49152	-0.01878	0.01312	-0.03438	0.00263	.	0.02201	0.24494
31	30	20	0	1	0	0	Lower	1	0.11682	0.04734	0.26041	-0.49845	-0.01822	0.01186	-0.03545	0.00247	.	0.02252	0.25194
32	78	20	1	0	0	1	Middle	1	0.21265	0.08522	0.43914	1.75962	0.14829	-0.05629	-0.05226	0.27409	.	-0.23708	3.28858
33	21	21	1	0	1	1	Middle	1	0.57330	0.33160	0.78442	1.05485	-0.03820	-0.01341	0.01876	0.16141	.	0.09840	1.16292
34	66	22	0	1	0	0	Lower	1	0.12310	0.05053	0.27023	-0.51256	-0.01696	0.00909	-0.03769	0.00212	.	0.02358	0.26653
35	16	22	0	0	1	1	Upper	1	0.47909	0.28142	0.68354	1.21315	0.10549	-0.05528	-0.11097	-0.13272	.	0.10445	1.52577

36	11	23	0	0	0	0	Upper	1	0.16403	0.06903	0.34176	-0.59860	-0.07437	0.01912	0.05175	0.04453	.	0.05261	0.36518
37	43	24	0	0	1	0	Upper	1	0.49395	0.29583	0.69399	-1.16715	-0.09070	0.03837	0.10385	0.12371	.	-0.09963	1.40958
38	81	25	1	0	0	0	Middle	1	0.23861	0.09829	0.47395	-0.73839	-0.03390	0.00011	0.01437	-0.08578	.	0.07126	0.56267
39	6	26	0	1	0	0	Lower	1	0.13653	0.05707	0.29233	-0.54184	-0.01385	0.00251	-0.04256	0.00128	.	0.02582	0.29819
40	9	26	1	0	0	1	Middle	1	0.24406	0.10093	0.48145	1.67950	0.10040	0.01042	-0.04459	0.27353	.	-0.22541	2.99610
41	13	27	0	0	0	0	Upper	1	0.18100	0.07840	0.36473	-0.63194	-0.07550	0.01099	0.05627	0.04777	.	0.05763	0.40740
42	70	28	0	0	0	0	Upper	1	0.18545	0.08079	0.37099	-0.64051	-0.07569	0.00867	0.05745	0.04860	.	0.05895	0.41866
43	68	30	0	0	0	0	Upper	1	0.19461	0.08559	0.38416	-0.65792	-0.07592	0.00366	0.05987	0.05028	.	0.06166	0.44206
44	8	31	1	0	0	1	Middle	1	0.27253	0.11409	0.52147	1.61246	0.06279	0.06227	-0.03847	0.27211	.	-0.21553	2.76692
45	55	31	0	0	1	0	Upper	1	0.54588	0.34358	0.73408	-1.25650	-0.06450	-0.01167	0.10973	0.12881	.	-0.11260	1.63491
46	95	32	0	1	0	1	Lower	1	0.15896	0.06701	0.33219	1.91785	0.03933	0.05488	0.26968	0.00210	.	-0.15624	3.85637
47	22	32	1	0	1	1	Middle	1	0.65079	0.40153	0.83810	0.92689	-0.06762	0.04432	0.02029	0.13975	.	0.08316	0.89443
48	1	33	0	0	0	0	Upper	1	0.20898	0.09281	0.40556	-0.68473	-0.07590	-0.00487	0.06363	0.05285	.	0.06591	0.47951
49	98	35	0	1	0	0	Lower	1	0.17126	0.07191	0.35531	-0.61294	-0.00330	-0.01840	-0.05573	-0.00145	.	0.03155	0.38332
50	33	35	1	0	0	0	Middle	1	0.29674	0.12441	0.55616	-0.83909	-0.01448	-0.04268	0.01425	-0.11409	.	0.08758	0.73309
51	2	35	0	0	0	0	Upper	1	0.21898	0.09759	0.42094	-0.70308	-0.07561	-0.01128	0.06624	0.05460	.	0.06888	0.50612
52	10	37	1	0	0	0	Middle	1	0.30931	0.12945	0.57423	-0.86030	-0.00925	-0.05360	0.01410	-0.12060	.	0.09116	0.77259
53	15	37	0	0	1	1	Upper	1	0.58965	0.37956	0.77143	1.02784	0.02561	0.04363	-0.07947	-0.09201	.	0.08650	1.08996
54	50	38	1	0	1	1	Middle	1	0.69019	0.43337	0.86648	0.86115	-0.07776	0.06620	0.02044	0.12804	.	0.07527	0.77207
55	85	39	1	0	0	0	Middle	1	0.32216	0.13439	0.59266	-0.88187	-0.00350	-0.06541	0.01390	-0.12743	.	0.09484	0.81410
56	57	39	0	0	1	0	Upper	1	0.60396	0.39037	0.78411	-1.36107	-0.02646	-0.08139	0.11555	0.13312	.	-0.12830	1.92752
57	49	40	1	0	1	1	Middle	1	0.70277	0.44296	0.87546	0.83992	-0.08033	0.07216	0.02040	0.12419	.	0.07271	0.73465
58	35	44	0	1	1	0	Lower	1	0.56602	0.28366	0.81117	-1.29210	0.19641	-0.15575	-0.26635	-0.02924	.	-0.20543	1.79985
59	77	46	0	1	0	0	Lower	1	0.22279	0.08875	0.45760	-0.70999	0.01863	-0.05929	-0.07693	-0.00689	.	0.04004	0.51997
60	69	46	0	0	0	0	Upper	1	0.28001	0.12255	0.51991	-0.81059	-0.06920	-0.05866	0.08206	0.06465	.	0.08725	0.67970
61	58	50	0	0	1	0	Upper	1	0.67901	0.43957	0.85086	-1.50754	0.04032	-0.19919	0.12156	0.13590	.	-0.15094	2.39781
62	84	51	0	1	0	0	Lower	1	0.24960	0.09562	0.51133	-0.75782	0.03299	-0.08525	-0.08886	-0.01037	.	0.04451	0.59705
63	74	52	0	0	0	1	Upper	1	0.31736	0.13470	0.58133	1.51506	0.13190	0.20421	-0.19742	-0.15128	.	-0.21258	2.45114
64	41	53	1	0	1	1	Middle	1	0.77682	0.49439	0.92531	0.71070	-0.08867	0.09691	0.01925	0.10008	.	0.05719	0.52764
65	65	59	0	1	0	1	Lower	1	0.29676	0.10555	0.60143	1.55875	-0.15028	0.32987	0.26448	0.04183	.	-0.12459	2.65895
66	4	60	0	0	0	0	Upper	1	0.37100	0.14908	0.66506	-0.96293	-0.04443	-0.15768	0.10593	0.07794	.	0.11613	0.98523
67	29	61	0	1	0	0	Lower	1	0.30932	0.10784	0.62395	-0.86032	0.07270	-0.15548	-0.11808	-0.01986	.	0.05478	0.78738
68	60	61	0	0	1	0	Upper	1	0.74582	0.47490	0.90494	-1.65512	0.12318	-0.34063	0.12464	0.13437	.	-0.17394	2.95470
69	48	65	0	1	1	0	Lower	1	0.70895	0.35026	0.91671	-1.57116	0.40560	-0.43105	-0.36607	-0.07048	.	-0.27791	2.80759
70	17	67	0	0	1	1	Upper	1	0.77815	0.49016	0.92752	0.70829	-0.04972	0.12152	-0.03561	-0.03751	.	0.05310	0.52488
71	51	68	1	0	1	1	Middle	1	0.84467	0.53631	0.96236	0.58104	-0.08489	0.10238	0.01660	0.07521	.	0.04190	0.35400
72	44	70	0	0	1	1	Upper	1	0.79317	0.49700	0.93704	0.68076	-0.05250	0.12278	-0.03250	-0.03380	.	0.05013	0.48565
73	52	74	0	0	1	1	Upper	1	0.81201	0.50546	0.94806	0.64535	-0.05521	0.12315	-0.02867	-0.02928	.	0.04631	0.43731

```
155        proc plot data=three;  plot yhat*Age='x' mean*Age='o' / overlay;
156             TITLE2 'Plot of observed means (o) and predicted values (p)';
157        run;

Logistic Regression - NKNW Example 14.3
Plot of observed means (o) and predicted values (p)
```




```
NOTE: 123 obs had missing values.
```

Logistic regression – how far can you take it? Interactions, polynomials, analysis of covariance. Yes!

Of course, the logistic is already a curve, so polynomials will alter the expected shape, including giving a quadratic shape to the log-odds dependent variable. Ditto for response surfaces; both of these are feasible within the context of logistic regression.

Other options that work much like PROC GLM or MIXED are options for a solution, NOINT and even STB, a standardized regression coefficient.

How about selection criteria (forward, backward, stepwise)? Yes, they are there in PROC LOGISTIC. There is also a "best subset" selection option, SELECTION=SCORE. Selection is based on the "score chi-square" value. Also available are the options usually associated with SAS stepwise applications, including the INCLUDE, SLENTRY, SLSTAY, START, STOP and BEST options.

Instead of plotting R^2, Adjusted R^2, Mallow's C_p or SSE_p or other aids to model selection, the LOGISTIC equivalent would be AIC_p, SBC_p,–2 log likelihood and score statistics.