Non Linear Regression : Maximum likelihood solution

Linear models : particularly linear in the parameters, additive parameters.

These models can include interactions, polynomials and transformations (logs, powers, inverses and square roots) of the dependent variables, but must have additive parameters.

Intrinsically Linear : These are models which are not linear in the parameters, but which can be transformed to linear models (eg. log-log and semi-log models).

Nonlinear : everything else.

How to solve : the criteria is usually the same as with least squares, we want to minimize the $SS_{deviations}$ of observations from the line, just like in Least Squares Regression,

$$Q = \sum_{i=1}^{n} \varepsilon_i^2 = \sum_{i=1}^{n} [Y_i - (\beta_0 + \beta_1 X_i)]^2$$

However, in least squares regression we start with a derivative of the model, and can then determine the unique solution that minimizes these $SS_{deviations}$. This is not usually possible with nonlinear models, and so there is no unique solution

$$Q = \sum_{i=1}^{n} \varepsilon_i^2 = \sum_{i=1}^{n} [Y_i - f(X_i, \gamma)]^2$$

where the partial derivative of Q is with respect to

$$\frac{\partial Q}{\partial \gamma_k} = \sum_{i=1}^{n} -2[Y_i - f(X_i, \gamma)]\left[\partial f(X_i, \gamma) / \partial \gamma_k\right]$$

So how do we estimate the parameters?

1) numerical search : pick some starting values, and iteratively try slightly different values until an optimal solution is found.

2) solving least squares normal equations of the derivatives - The first derivatives of the equation are taken and set equal to zero. These can sometimes be solved as normal equations for a solution.

In most cases some numerical search is necessary.

There are a number of search routines available; your text discusses one, the Gauss-Newton which employs a linear appproximation of the nonlinear function (Taylor series approximation)

linear techniques can then be used to obtain parameter estimates.

then check to see if there is improvement in the fit, there should be

but this fit was only an approximation, now substitute in the new estimates and re-approximate

This is done iteratively until some criteria is met, usually based on the change in successive SSE or successive $b_k$

others are available (SAS has Gauss, Marquardt, Newton, Gradient and DUD (the latter **D**oesn't **U**se **D**erivatives)

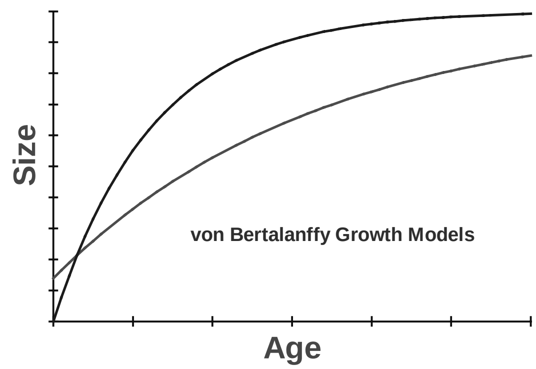
Even if the first derivatives do not provide solvable normal equations, they still can be used to determine the direction of search, and to determine when to stop the search.

Note: SAS versions after version 6 do not require derivatives; SAS will obtain the derivatives as part of the program if they are omitted.

Growth curves

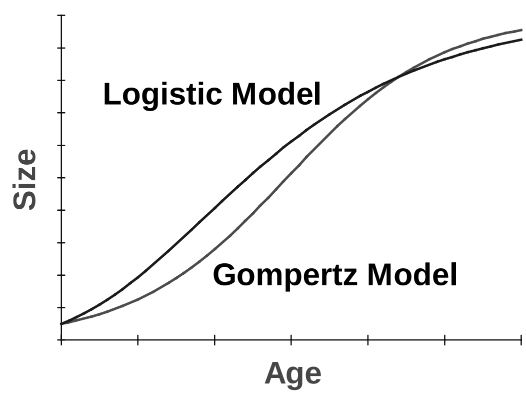
von Bertalanffy (neg exp) $$l_t = L_\infty \left(1 - e^{-k(t-t_0)}\right)$$

or $$l_t = L_\infty - \left(L_\infty - l_0\right) e^{-kt}$$




Gompertz $$l_t = L_0 e^{L'_\infty \left(1 - e^{-kt}\right)}$$

Logistic $$l_t = \frac{L_\infty}{1 + \left(\frac{L_\infty - L_0}{L_0}\right) e^{-kt}}$$




Richard's 4 parm model $$l_t = L_\infty^{1-m} - \left[\left(L_\infty^{1-m} - l_0^{1-m}\right) e^{-(1-m)kt}\right]^{1-m}$$

**or** $$l_t = L_\infty \left(1 - e^{-k(t-t_0)}\right)^p$$

Inferences from Nonlinear models

1) MSE is not normal, but will be approximately normal for large n

2) Under these conditions we can use MSE derived standard errors for tests and confidence intervals.

3) Testing hypotheses is better handled by fitting full and reduced models, and using a test similar to the General Linear Test

The textbook describes this as an approximate test , but a likelihood ratio test (very similar but 2 instead of F) is better

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NOTE: An intrinsically linear function like $Y_i = \beta_0 e^{\beta_1 X_i}$

can be fitted linearly (with multiplicative error) by taking the log of

$$Y_i = \beta_0 e^{\beta_1 X_i} \varepsilon_i$$

it can also be fitted directly as a nonlinear function, but the model is

$$Y_i = \beta_0 e^{\beta_1 X_i} + \varepsilon_i$$

since NLIN will treat the error as additive.