Three factor multiple regression from Snedecor and Cochran (1967), table 13.10.1, page 405.

Y = estimated plant available phosphorus in the soil (20 C)
X_1 = inorganic phosphorus
X_2 = organic phosphorus soluble in K_2CO_3 and hydrolized by hypobromite
X_3= organic phosphorus soluble in K_2CO_3 and NOT hydrolized by hypobromite

All least squares regression analyses start with the same three matrices.

X =

1	0.4	53	158
1	0.4	23	163
1	3.1	19	37
1	0.6	34	157
1	4.7	24	59
1	1.7	65	123
1	9.4	44	46
1	10.1	31	117
1	11.6	29	173
1	12.6	58	112
1	10.9	37	111
1	23.1	46	114
1	23.1	50	134
1	21.6	44	73
1	23.1	56	168
1	1.9	36	143
1	26.8	58	202
1	29.9	51	124

Y =

64
60
71
61
54
77
81
93
93
51
76
96
77
93
95
54
168
99

X´X =

18	215	758	2214
215	4321.02	10139.5	27645
758	10139.5	35076	96598
2214	27645	96598	307894

X´Y =

1463
20706.2
63825
187542

Y´Y = [131299]

Create a fully augmented matrix of the form;

X´X	X´Y	I
(X´Y)´	Y´Y	0

The resulting matrix contains;

X_0	X_1	X_2	X_3	$X'Y$	c_0	c_1	c_2	c_3
n	ΣX_1	ΣX_2	ΣX_3	ΣY	1	0	0	0
ΣX_1	ΣX_1^2	ΣX_1X_2	ΣX_1X_3	ΣX_1Y	0	1	0	0
ΣX_2	ΣX_1X_2	ΣX_2^2	ΣX_2X_3	ΣX_2Y	0	0	1	0
ΣX_3	ΣX_1X_3	ΣX_2X_3	ΣX_3^2	ΣX_3Y	0	0	0	1
ΣY	ΣX_1Y	ΣX_2Y	ΣX_3Y	ΣY^2	0	0	0	0

Numerically for this problem given previously the matrix is;

X_0	X_1	X_2	X_3	$X'Y$	c_0	c_1	c_2	c_3
18	215	758	2214	1463	1	0	0	0
215	4321.02	10139.5	27645	20706.2	0	1	0	0
758	10139.5	35076	96598	63825	0	0	1	0
2214	27645	96598	307894	187542	0	0	0	1
1463	20706.2	63825	187542	131299	0	0	0	0

The first step (divide row 1 by value$_{1,1}$) in the sweepout technique produces,

X_0	X_1	X_2	X_3	$X'Y$	c_0	c_1	c_2	c_3
1	11.944444	42.111111	123	81.277778	0.055556	0	0	0
215	4321.02	10139.5	27645	20706.2	0	1	0	0
758	10139.5	35076	96598	63825	0	0	1	0
2214	27645	96598	307894	187542	0	0	0	1
1463	20706.2	63825	187542	131299	0	0	0	0

And after sweeping out the first column (subtracting a multiple of row 1 from all other rows) we have;

X_0	X_1	X_2	X_3	$X'Y$	c_0	c_1	c_2	c_3
1	11.944444	42.111111	123	81.277778	0.055556	0	0	0
0	1752.964444	1085.611111	1200	3231.477778	-11.944444	1	0	0
0	1085.611111	3155.777778	3364	2216.444444	-42.111111	0	1	0
0	1200	3364	35572	7593	-123	0	0	1
0	3231.477778	2216.444444	7593	12389.61111	-81.277778	0	0	0

We start the second column sweep by dividing row 2 by value$_{2,2}$,

X_0	X_1	X_2	X_3	$X'Y$	c_0	c_1	c_2	c_3
1	11.944444	42.111111	123	81.277778	0.055556	0	0	0
0	1	0.6193	0.684555	1.843436	-0.006814	0.00057	0	0
0	1085.611111	3155.777778	3364	2216.444444	-42.111111	0	1	0
0	1200	3364	35572	7593	-123	0	0	1
0	3231.477778	2216.444444	7593	12389.61111	-81.277778	0	0	0

and finish sweeping the second column to obtain;

X_0	X_1	X_2	X_3	X´Y	c_0	c_1	c_2	c_3
1	0	34.713915	114.823375	59.258959	0.136943	-0.006814	0	0
0	1	0.6193	0.684555	1.843436	-0.006814	0.00057	0	0
0	0	2483.458674	2620.839842	215.189831	-34.713915	-0.6193	1	0
0	0	2620.839842	34750.53439	5380.87679	-114.823375	-0.684555	0	1
0	0	215.189831	5380.87679	6432.588616	-59.258959	-1.843436	0	0

The third column starts with,

X_0	X_1	X_2	X_3	X´Y	c_0	c_1	c_2	c_3
1	0	34.713915	114.823375	59.258959	0.136943	-0.006814	0	0
0	1	0.6193	0.684555	1.843436	-0.006814	0.00057	0	0
0	0	1	1.055318	0.086649	-0.013978	-0.000249	0.000403	0
0	0	2620.839842	34750.53439	5380.87679	-114.823375	-0.684555	0	1
0	0	215.189831	5380.87679	6432.588616	-59.258959	-1.843436	0	0

and after being swept out produces,

X_0	X_1	X_2	X_3	X´Y	c_0	c_1	c_2	c_3
1	0	0	78.189139	56.251024	0.622176	0.001843	-0.013978	0
0	1	0	0.030996	1.789774	0.001843	0.000725	-0.000249	0
0	0	1	1.055318	0.086649	-0.013978	-0.000249	0.000403	0
0	0	0	31984.71367	5153.782984	-78.189139	-0.030996	-1.055318	1
0	0	0	5153.782984	6413.942579	-56.251024	-1.789774	-0.086649	0

Finally the fourth column in the X´X matrix is started and swept out,

X_0	X_1	X_2	X_3	X´Y	c_0	c_1	c_2	c_3
1	0	0	78.189139	56.251024	0.622176	0.0018428	-0.0139781	0
0	1	0	0.030996	1.789774	0.001843	0.0007249	-0.0002494	0
0	0	1	1.055318	0.086649	-0.013978	-0.0002494	0.0004027	0
0	0	0	1	0.161133	-0.002445	-0.0000010	-0.0000330	0.000031
0	0	0	5153.782984	6413.942579	-56.251024	-1.7897741	-0.0866492	0

and the final result is;

X_0	X_1	X_2	X_3	X´Y	c_0	c_1	c_2	c_3
1	0	0	0	43.652198	0.813316	0.0019185	-0.0113982	-0.002445
0	1	0	0	1.78478	0.001919	0.0007249	-0.0002483	-0.000001
0	0	1	0	-0.083397	-0.011398	-0.0002483	0.0004375	-0.000033
0	0	0	1	0.161133	-0.002445	-0.0000010	-0.0000330	0.000031
0	0	0	0	5583.499658	-43.652198	-1.7847797	0.0833971	-0.161133

The resulting matrix is of the form;

$$\left[\begin{array}{c|c|c} I & B & (X'X)^{-1} \\ \hline 0 & SSE & -B' \end{array}\right]$$

and contains the values

$$\begin{array}{cccc|c|cccc} X_0 & X_1 & X_2 & X_3 & X'Y & c_0 & c_1 & c_2 & c_3 \end{array}$$

$$\left[\begin{array}{cccc|c|cccc} 1 & 0 & 0 & 0 & b_0 & c_{00} & c_{01} & c_{02} & c_{03} \\ 0 & 1 & 0 & 0 & b_1 & c_{10} & c_{11} & c_{12} & c_{13} \\ 0 & 0 & 1 & 0 & b_2 & c_{20} & c_{21} & c_{22} & c_{23} \\ 0 & 0 & 0 & 1 & b_3 & c_{30} & c_{31} & c_{32} & c_{33} \\ \hline 0 & 0 & 0 & 0 & SSE & -b_0 & -b_1 & -b_2 & -b_3 \end{array}\right]$$

The solution to the regression equation is then,

$$Y_i = 43.652 + 1.785X_{1i} - 0.083X_{2i} + 0.161X_{3i} + e$$

The sums of squares are given by the sequential reduction in the YY matrix

MATRIX	Y´Y VALUE	INTERPRETATION of the REPLACEMENT VALUE	DIFFERENCE from PREVIOUS VALUE	INTERPRETATION of the DIFFERENCE
Original	**131299**	**ΣY^2 (uncorrected)**		
Col 1 sweep	**12389.6111**	**$\Sigma Y^2-(\Sigma Y)^2/n = SSY\|X_0$**	**118909.3840**	**$(Y)^2/n$ = C.F.**
Col 2 sweep	**6432.5886**	**$SSY\|X_0,X_1$**	**5957.0225**	**$SeqSSX_1$**
Col 3 sweep	**6413.9426**	**$SSY\|X_0,X_1,X_2$**	**18.6460**	**$SeqSSX_2$**
Col 4 Sweep	**5583.4997**	**$SSY\|X_0,X_1,X_2,X_3$ = SSE**	**830.4429**	**$SeqSSX_3$**

Partial sums of squares, or fully adjusted sums of squares, are given by

$$\text{PARTIAL SS} = \frac{b_k}{c_{kk}}$$

Partial $SSX_1 = b_1^2/c_{11} = 1.7848^2 / 0.0007249 = 4394.1523$

Partial $SSX_2 = b_2^2/c_{22} = 0.08340^2 / 0.0004375 = 15.8979$

Partial $SSX_3 = b_3^2/c_{33} = 0.1611^2 / 0.00003127 = 830.4429$

Recall that I number the positions in the X´X matrix differently, from k = 0, 1, ... , p (where p is the number of parameters excluding the intercept) instead of starting at 1 as other matrices. This is done in order to be able to associate the matrix position with the regression coefficient subscript.