General Linear Hypothesis Test Approach (GLHT)

Given a Regression Model with all variables of interest

$$Y_i = b_0 + b_1 X_i + e_i \quad = \quad \overline{Y}_i + b_1(X_i - \overline{X}) + e_i$$

we will call this the FULL model

Given a second Regression Model with parameters of interest which are a subset of the FULL model; for the SLR we are testing the hypothesis that $\beta_1 = 0$, then

$$Y_i = b_0 + \epsilon_i = \overline{Y}_i + \epsilon_i$$

we will call this the REDUCED model

Both models have a total of “n" observations

The reduced model has “p" parameters including the intercept (=1 for SLR)

The full model has “p + q" parameters i*including the intercept (=2 for SLR)

Our objective is to perform a test of the DIFFERENCE between the two models.

1) If the models are DIFFERENT, then the full model is BETTER since it has more information.

2) If the models are NOT DIFFERENT, then the reduced model is better, because it fits equally as well with fewer degrees of freedom.

FIT BOTH MODELS

from each take the d.f. Error and SSError

Perform the General Linear Hypothesis test as follows

Model	d.f	SS	MS	F
Reduced (Error)	p+q	SSE_{Reg}		
Full (Error)	p	SSE_{Full}		
Difference	q	SS_{Diff}	MS_{Diff}	$\frac{MS_{Diff}}{MSE_{Full}}$
Full (Error)	n-p-q	SSE_{Full}	MSE_{Full}	

1) The Reduced Model will always have more degrees of freedom in the error because it has fewer terms and fewer d.f. in the model

2) The Reduced Model will always have a larger (or equal) Sum of Squares Error because it cannot fit as well with fewer parameters

3) Therefore, the $d.f._{Diff}$ and SS_{Diff} will always be positive (or zero)

4) To test the DIFFERENCE, what do we use as an error term?
   a) Either there is no difference between the models, and it makes no difference which error term is used, OR ...
   b) There is a difference in the models, in which case the FULL model is better

SO WE USE THE FULL MODEL ERROR TERM

For the SLR, the SSE(full) is simply the SSE for the SLR
the SSE(reduced) is the CORRECTED SSTotal

The difference then has one degree of freedom, and is equal to the SSRegression

Model	d.f	SS	MS	F
Reduced (Error)	n-1	SSTotal		
Full (Error)	n-2	SSE		
Difference	1	SS_{diff}=SS_{Reg}	MSReg	$\frac{MSReg}{MSError}$
Full (Error)	n-2	SSE	MSE	

Which is the same as the test of the model we saw in our ANOVA table. This is another way of viewing the test that we have done. However, this test is readily generalized to any full and reduced model, where the reduced model has a subset of the parameter estimates of the full model.