POWER $= 1 - \beta$ in testing regression coefficients.

If we reject a null hypothesis, we need not concern ourselves with power. If we reject H_0, we have a $(1-\alpha)100\%$ chance of having made an error (called TYPE I error or α error) in saying a difference exists (eg. $\beta_1 \neq 0$) when in fact a difference did not exist. We set α, and we know α, if the assumptions are met.

If we fail to reject the null hypothesis, we cannot make a TYPE I or α error. However, it is also possible that we have made an error in not demonstrating a difference exists when in fact one does exist. This error is called TYPE II or β error (not to be confused with regression coefficients "β").

A β error can be made only when H_1 is true, and the probability of making this error can be calculated only when we know the real difference between the hypothesized and true value of the regression coefficient. In practice this cannot be known, so we never really know the probability of TYPE II error.

For this reason, we never "accept" the null hypothesis, we only state that we cannot reject or that the null hypothesis is consistent with the data.

However, if we are willing to "guess" at the difference between the hypothesized and true values of the regression coefficients we can calculate a value for β, the probability of a TYPE II error and POWER $= 1 - \beta$.

$$\text{Power} = \text{P}\{|\text{ observed t }| > \text{tabular } t_{1-\frac{\alpha}{2}, n-2} \mid \delta\}$$

where δ is called the *noncentrality parameter*, and is a standardized measure of how far the value of the true parameter (β_1) departs from the value of the hypothesized parameter β_{1_0} .

$$\delta = \frac{(\beta_1 - \beta_{1_0})}{\sigma_{\beta_1}}$$

In practice, σ_{b_1} is estimated by s_{b_1} and β_1 is estimated by b_1.

However, b_1 is not the true value of the parameter, only an estimate. All we can state in calculating power is that "if b_1 were the true value then the power would be ...", or "if the difference between the true value and the hypothesized value is really $(\beta_1 - \beta_{1_0})$ then the power is ...".

Example of Power Calculation from Power Chart : Take a generic example.

X	Y
1	11
2	9, 10
3	17, 18, 8
4	7
5	16, 19
7	12

Model Crossproducts X′X X′Y Y′Y

X′X	INTERCEP	X	Y
INTERCEP	10	35	127
X	35	151	465
Y	127	465	1789

X′X Inverse, Parameter Estimates, and SSE

	INTERCEP	X	Y
INTERCEP	0.5298245614	-0.122807018	10.18245614
X	-0.122807018	0.0350877193	0.7192982456
Y	10.18245614	0.7192982456	161.35438596

Dependent Variable: Y
Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Value	Prob>F
Model	1	14.74561	14.74561	0.731	0.4174
Error	8	161.35439	20.16930		
C Total	9	176.10000			

Parameter Estimates

Variable	DF	Parameter Estimate	Standard Error	T for H0: Parameter=0	Prob > |T|
INTERCEP	1	10.182456	3.26897378	3.115	0.0143
X	1	0.719298	0.84124591	0.855	0.4174

Calculate power where β_1 is hypothesized to be 0. P = 0.4174.

$$\hat{\delta} = \frac{(b_1 - \beta_{1_0})}{s_{b_1}} = \frac{(0.7193 - 0)}{0.8412} = 0.855 \text{ which is the t statistic}$$

Now examine power table (α=0.05) with $\hat{\delta}$ = 0.855 and 8 df.
Power is only about 8%, or P(TYPE II error) = 1-0.08 = 0.92

Calculate power where β_0 is hypothesized to be 0. P = 0.0143.

Now examine power table (α=0.05) with $\hat{\delta}$ = 3.115 and 8 df.
Power is only about 74%, or P(TYPE II error) = 1-0.74 = 0.26