Properties of the FITTED REGRESSION LINE - see proofs NWK Ch. 2, pg 47

A Simple Linear Regression line has the following properties

1) $\mathbf{\Sigma\hat{Y}_i = \Sigma Y_i}$ so both have the same mean when divided by n

$\hat{Y}_i = \bar{Y} + \hat{\beta}_1(X_i - \bar{X})$ and since $\sum_{i=1}^{n}(X_i - \bar{X}) = 0$

$\Sigma\hat{Y}_i = n\bar{Y} \Rightarrow \Sigma\hat{Y}_i = \Sigma Y_i$ NWK 2.18

2) $\mathbf{\sum_{i=1}^{n} e_i = 0}$ NWK 2.17

$e_i = Y_i - \hat{Y}_i$

$$\sum_{i=1}^{n} e_i = \sum_{i=1}^{n}(Y_i - \hat{Y}_i) = \sum_{i=1}^{n} Y_i - \sum_{i=1}^{n}\hat{Y}_i = 0$$

3) $\mathbf{\sum_{i=1}^{n} e_i^2}$ **is a minimum**

This condition is met by the derivation of the parameter estimates

4) $\mathbf{\sum_{i=1}^{n} e_i X_i = 0}$ NWK 2.19

So the sum of the weighted residuals (weight = X_i) is zero

Proof; $\sum_{i=1}^{n} e_i X_i = \Sigma(Y_i - \hat{Y}_i)X_i = \Sigma Y_i X_i - \Sigma\hat{Y}_i X_i =$

$\Sigma Y_i X_i - \Sigma[\bar{Y} + b_1(X_i - \bar{X})] X_i =$

$\Sigma Y_i X_i - \frac{(\Sigma X_i)(\Sigma Y_i)}{n} - b_1\Sigma(X_i - \bar{X}) X_i =$

$\Sigma Y_i X_i - \frac{(\Sigma X_i)(\Sigma Y_i)}{n} - \Sigma Y_i X_i - \frac{(\Sigma X_i)(\Sigma Y_i)}{n} = 0$

5) $\sum_{i=1}^{n} e_i \hat{Y}_i = 0$ NWK 2.20

So the sum of the weighted residuals (weight = Y_i) is zero

Proof; $\sum_{i=1}^{n} e_i \hat{Y}_i = \Sigma e_i [\bar{Y} + b_1(X_i - \bar{X})] =$

$\Sigma e_i \bar{Y} - b_1 \Sigma e_i (X_i - \bar{X}) = \Sigma e_i \bar{Y} - b_1 \Sigma e_i X_i - b_1 \bar{X} \Sigma e_i = 0$

5) $\hat{Y}_i (Y_i - \hat{Y}_i) = Y_i \hat{Y}_i - \hat{Y}_i \hat{Y}_i \Rightarrow \Sigma \hat{Y}_i (Y_i - \hat{Y}_i) = \Sigma Y_i \hat{Y}_i - \Sigma \hat{Y}_i \hat{Y}_i$

$= \Sigma \hat{Y}_i (\bar{Y}_i + \hat{\beta}_i (X_i - \bar{X})) - \Sigma(\bar{Y}_i^2 + 2\hat{\beta}_i (X_i - \bar{X}) + \hat{\beta}_i^2 (X_i - \bar{X})^2)$

$= n\bar{Y}_i^2 + \hat{\beta}_i Y_i (X_i - \bar{X})) - n\bar{Y}_i^2 - \hat{\beta}_i^2 (X_i - \bar{X})^2 = 0$

For the GENERAL CASE - MLR

1) $\sum_{i=1}^{n} \hat{Y}_i = \sum_{i=1}^{n} Y_i$

2) $\sum_{i=1}^{n} e_i = 0$ if there is an intercept

3) $\sum_{i=1}^{n} e_i X_{1i} = \sum_{i=1}^{n} e_i X_{2i} = \ldots = \sum_{i=1}^{n} e_i X_{ki} = 0$

4) $\sum_{i=1}^{n} e_i \hat{Y}_{1i} = \sum_{i=1}^{n} e_i \hat{Y}_{2i} = \ldots = \sum_{i=1}^{n} e_i \hat{Y}_{ki} = 0$

General proof:
a) $\hat{Y} = HY \qquad 1'\hat{Y} = 1'HY = 1'Y$
b) $e = (I - H)Y \Rightarrow X'e = X'(I - H)Y$
c) $\hat{Y}'(I - H)Y = Y'H(I - H)Y = Y$