Simple Linear Regression

Data: Neter, Wasserman & Kuttner (1989), Page 57, Problem 2.19.

X	Y	$\bar{Y}$
0	8, 9, 11, 12	10
1	13, 15, 16	14.67
2	17, 19	18
3	22	22

Intermediate Calculations

$\Sigma X_i = 10$; $\Sigma X_i^2 = 20$; $\Sigma(X_i-\bar{X})^2 = 10$; $\bar{X} = 1$

$\Sigma Y_i = 142$; $\Sigma Y_i^2 = 2194$; $\Sigma(Y_i-\bar{Y})^2 = 177.6$; $\bar{Y} = 14.2$

$\Sigma X_i Y_i = 182$; $\Sigma(X_i-\bar{X})(Y_i-\bar{Y}) = 40$

Algebraic form $10b_0 + 10b_1 = 142$

$10b_0 + 20b_1 = 182$

Normal Equations (NE) Matrix form $\begin{bmatrix} \mathbf{10} & \mathbf{10} \\ \mathbf{10} & \mathbf{20} \end{bmatrix} \begin{bmatrix} \mathbf{b_0} \\ \mathbf{b_1} \end{bmatrix} = \begin{bmatrix} \mathbf{142} \\ \mathbf{182} \end{bmatrix}$

$$b_1 = \frac{\Sigma(X_i-\bar{X})(Y_i-\bar{Y})}{\Sigma(X_i-X)^2} = \frac{\Sigma X_i Y_i - (\Sigma X_i \Sigma Y_i)/n}{\Sigma X_i^2 - \Sigma(X_i)^2/n} = 4.0$$

$$b_0 = \bar{Y} - b_1\bar{X} = 10.2$$

Sums of Squares (SS) Regression $= \Sigma(\hat{Y}_i - \bar{Y})^2 = b_1^2\, \Sigma(X_i - \bar{X})^2 = 160$

Total (Corrected) $= \Sigma(Y_i - \hat{Y}_i)^2 = \Sigma Y_i^2 - \frac{(\Sigma Y)^2}{n} = 2194 - \frac{142^2}{10} = 177.6$

Residual = SSTotal - SSRegression = 177.6 - 160 = 17.6

ANOVA TABLE

SOURCE	d.f.	SS	MS	F
Regression	1	160.0	160.0	72.727
Residual or Error	8	17.6	2.20	
Total	9	177.6		

$F_{0.05,\,1,10\,df} = 5.32$

WE CONCLUDE THAT THERE IS EVIDENCE THAT $\beta_1 \neq 0$.

Error Estimates: $S^2 = \frac{\text{SSError}}{\text{df Error}} = \text{MSE} = \frac{17.6}{8} = 2.20$

$$S_{b_0} = \sqrt{S^2\left[\frac{1}{n} + \frac{\bar{X}^2}{\Sigma(X_i-X)^2}\right]} = 0.6633 \text{ and } S_{b_1} = \sqrt{\frac{S^2}{\Sigma(X_i-X)^2}} = 0.4690$$