Three factor multiple regression from Snedecor and Cochran (1967), table 13.10.1, page 405.

Y = estimated plant available phosphorus in the soil (20 C)
X_1 = inorganic phosphorus
X_2 = organic phosphorus soluble in K_2CO_3 and hydrolized by hypobromite
X_3= organic phosphorus soluble in K_2CO_3 and NOT hydrolized by hypobromite

All least squares regression analyses start with the same three matrices.

$$X = \begin{bmatrix} 1 & 0.4 & 53 & 158 \\ 1 & 0.4 & 23 & 163 \\ 1 & 3.1 & 19 & 37 \\ 1 & 0.6 & 34 & 157 \\ 1 & 4.7 & 24 & 59 \\ 1 & 1.7 & 65 & 123 \\ 1 & 9.4 & 44 & 46 \\ 1 & 10.1 & 31 & 117 \\ 1 & 11.6 & 29 & 173 \\ 1 & 12.6 & 58 & 112 \\ 1 & 10.9 & 37 & 111 \\ 1 & 23.1 & 46 & 114 \\ 1 & 23.1 & 50 & 134 \\ 1 & 21.6 & 44 & 73 \\ 1 & 23.1 & 56 & 168 \\ 1 & 1.9 & 36 & 143 \\ 1 & 26.8 & 58 & 202 \\ 1 & 29.9 & 51 & 124 \end{bmatrix} \quad Y = \begin{bmatrix} 64 \\ 60 \\ 71 \\ 61 \\ 54 \\ 77 \\ 81 \\ 93 \\ 93 \\ 51 \\ 76 \\ 96 \\ 77 \\ 93 \\ 95 \\ 54 \\ 168 \\ 99 \end{bmatrix}$$

All least squares regression analyses start with the same three matrices.

$$X'X = \begin{bmatrix} 18 & 215 & 758 & 2214 \\ 215 & 4321.02 & 10139.5 & 27645 \\ 758 & 10139.5 & 35076 & 96598 \\ 2214 & 27645 & 96598 & 307894 \end{bmatrix} \quad X'Y = \begin{bmatrix} 1463 \\ 20706.2 \\ 63825 \\ 187542 \end{bmatrix}$$

$$Y'Y = \begin{bmatrix} 131299 \end{bmatrix}$$

Create a fully augmented matrix of the form;

$$\begin{bmatrix} X'X & X'Y & I \\ (X'Y)' & Y'Y & 0 \end{bmatrix}$$