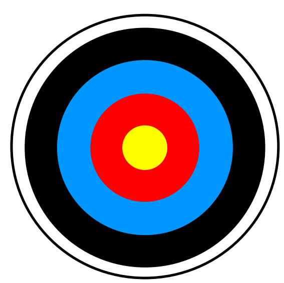
Statistical Techniques II

EXST7015

Post-ANOVA Tests
Contrasts



Overview of ANOVA

- **Recall that we are testing for differences among indicator variables.**
  - **The treatments may be fixed or random.**
  - **H_0: $\mu_1 = \mu_2 = \mu_3 = ... = \mu_k$ for fixed effects.**
  - **H_0: $\sigma^2_\tau = 0$ for random effects.**
- **Assume $e_i \sim NIDrv(0,\sigma^2)$. Remember that this covers 3 separate assumptions.**
- **Also, assume no block "interactions" for the RBD.**

Overview *(continued)*

- **Every analysis can be expressed as a model with appropriate notation and subscripting.**
- **CRD : $Y_{ij} = \mu + \tau_i + \varepsilon_{ij}$**
- **For the moment we will be concerned only with examining for differences among the treatment levels.**
- **We will assume that we have already detected a significant difference among treatments levels with ANOVA.**

Overview *(continued)*

- Treatments levels may be fixed or random. Determining appropriate tests depends on recognizing correctly.
- With random effects we are probably not interested in individual treatment levels. We are likely to be interested in the variability among the treatment levels and the distribution of the levels.
- With fixed effects we will probably want to compare individual levels.

Post ANOVA tests

- **Having rejected the Null hypothesis we wish to determine how the treatment levels interrelate. This is the "post-ANOVA" part of the analysis.**
- **These tests fall into two general categories.**
  - **Post hoc tests (LSD, Tukey, Scheffé, Duncan's, Dunnett's, etc.)**
  - ***A priori* tests or pre-planned comparisons (contrasts)**

Post ANOVA *(continued)*

- ***A priori*** **tests are better. These are tests that the researcher plans on doing before they gather data, and if we dedicate 1 d.f. to each one we generally feel comfortable doing each at some specified level of alpha.**

Post ANOVA *(continued)*

- However, since multiple tests do entail risks of higher experiment wide error rates, it would not be unreasonable to apply some technique, like Bonferroni's adjustment, to insure an experimentwise error rate of the desired level of alpha (α).
- So how might we do these "post hoc" tests?

Post ANOVA *(continued)*

- The simplest approach would be to do pairwise test of the treatments using something like the two-sample t-test.
- This tests examines the null hypothesis
  - H_0: $\mu_1 = \mu_2$ or H_0: $\mu_1 - \mu_2 = 0$,
  - against the alternative
  - H_a:μ_1-$\mu_2 \neq 0$, or H_a:μ_1-$\mu_2 \geq 0$ or H_a:μ_1-$\mu_2 \leq 0$.
-

Post-ANOVA tests

- **The test we have seen so far are often (usually?) done with no *a priori* tests in mind. We do not have certain comparisons in mind before doing the experiment, we want to examine many, or all, levels of the treatments for differences from one another.**
- **The experimentwise error rate is intended to allow this (except for the LSD).**

Post-ANOVA tests *(continued)*

- **However, sometimes we do have some particular comparisons in mind when we do an experiment.**
- **When we want some lesser number of comparisons, and they are determined *a priori* (without looking at the data), then we can use a less stringent criteria.**

Post-ANOVA tests *(continued)*

- **We generally feel comfortable with one test per degree of freedom at some specified level of alpha (α), just as we did in regression (looking at each regression coefficient with an α level of error).**
- **This is the case with *a priori* contrasts.**

Contrasts

- Recall out discussion of linear combinations from Multiple Regression
  - $A_i = aX_i + bY_i + cZ_i$
  - $Var(A_i)=a*Var(X_i) + b*Var(Y_i) + c*Var(Z_i)$ +2*Covariances
  - $Var(A_i) = a^2\sigma^2_{Xi} + b^2\sigma^2_{Yi} + c^2\sigma^2_{Zi} +2(ab\sigma_{Xi,Yi} + ac\sigma_{Xi,Zi} + bc\sigma_{Yi,Zi})$
- If the variables are independent we can assume all covariances are zero.

Contrasts *(continued)*

- In multiple regression we did not assume that the regression coefficients were independent. However, in ANOVA we do consider the levels of a treatment to be independent.
-

Contrasts *(continued)*

- **Suppose we want to test the mean of two groups against the mean of 3 other groups.**
- **H_0: $(\mu_1 + \mu_2)/2 = (\mu_3 + \mu_4 + \mu_5)/3$**
- **H_0: $(\mu_1 + \mu_2)/2 - (\mu_3 + \mu_4 + \mu_5)/3 = 0$**
- **H_0: ${}^1/_2\mu_1 + {}^1/_2\mu_2 - {}^1/_3\mu_3 - {}^1/_3\mu_4 - {}^1/_3\mu_5 = 0$**
-

Contrasts *(continued)*

- The variance of a mean is σ^2/n. In ANOVA all of the σ^2 are equal to MSE. The n may or may not be equal. Since we do not need the covariances we can calculate the variance as a linear combination,
- H_0: $^1/_2\mu_1 + ^1/_2\mu_2 - ^1/_3\mu_3 - ^1/_3\mu_4 - ^1/_3\mu_5 = 0$
  - Var: $(^1/_2)^2$*MSE($1/n_1$) + $(^1/_2)^2$*MSE($1/n_2$) + $(^{-1}/_3)^2$*MSE($1/n_3$) + $(^{-1}/_3)^2$*MSE($1/n_4$) +$(^{-1}/_3)^2$*MSE($1/n_5$) =
  - MSE($1/4n_1+1/4n_2+1/9n_3+1/9n_4+1/9n_5$)

Contrasts *(continued)*

- **Note that we already saw this for the two sample t-test as**
  - **$t = (\bar{Y}_1 - \bar{Y}_2) / \sqrt{(MSE((1/n_1+1/n_2)))}$.**
  - **If the design is balanced we can simplify this to $t = (\bar{Y}_1 - \bar{Y}_2)/\sqrt{(2MSE/n)}$.**
- **Of course, this is still true for 2 means.**
- **We also saw another type of application for linear combinations.**

Contrasts *(continued)*

- If you want to test an hypothesis between two or more independent estimates like,
  - H_0: $\mu_1 = 0.5\mu_2$ or $\mu_1 - 0.5\mu_2 = 0$
- We note that since these are independent, the variance for this t-test will be
  - $1^2 Var(\mu_1) + 0.5^2 Var(\mu_2)$
  - $MSE(1/n_1+0.25/n_2)$
- This is for two means.

Contrasts *(continued)*

- For more means we have a t-test that looks like the following.
  - H_0: $(\mu_1 + \mu_2)/2 - (\mu_3 + \mu_4 + \mu_5)/3 = 0$
  - H_0: $\frac{1}{2}\mu_1 + \frac{1}{2}\mu_2 - \frac{1}{3}\mu_3 - \frac{1}{3}\mu_4 - \frac{1}{3}\mu_5 = 0$
-
- And the variance is
  - $MSE(1/4n_1 + 1/4n_2 + 1/9n_3 + 1/9n_4 + 1/9n_5)$

Contrasts *(continued)*

- The general formula,
  - $H_0: m_1\mu_1+m_2\mu_2+m_3\mu_3+m_4\mu_4+m_5\mu_5=0$
  - Linear combo: $m_i\mu_3$, where we will call the m_i the "multipliers", estimate μ_i with $\bar{Y}_i$ and the variance with MSE.
  - Test statistic: $\Sigma m_i \ \bar{Y}_i$
  - Variance: $(MSE/n)\Sigma m^2_i$
    - For balanced data with all n equal.

Contrasts *(continued)*

- So all we need are the multipliers. These are often called the "contrast".
- I prefer these as integers. Note for the example we have examined.
- H_0: $\frac{1}{2}\mu_1 + \frac{1}{2}\mu_2 - \frac{1}{3}\mu_3 - \frac{1}{3}\mu_4 - \frac{1}{3}\mu_5 = 0$
- If we multiply by 6 we get,
- H_0: $3\mu_1 + 3\mu_2 - 2\mu_3 - 2\mu_4 - 2\mu_5 = 0$
- The multipliers are 3, 3, -2, -2, -2 instead of $\frac{1}{2}$, $\frac{1}{2}$, $-\frac{1}{3}$, $-\frac{1}{3}$, $-\frac{1}{3}$.

Contrasts *(continued)*

- Coming up with the multipliers.
- The multipliers should be *a priori* contrasts of interest to the investigator.
- As such, there are no "right or wrong" contrasts as long as they conform to one basic rule. The multipliers must add to zero.
-

Contrasts *(continued)*

- Lets suppose we are measuring hemoglobin concentrations in the blood, and are interested in the following animals.

- Cats
- Dogs
- Humans
- Whales (Pilot)
- Fish (perch)
- Sharks (Dogfish)
- Birds (Chicken)

Contrasts *(continued)*

- Now lets come up with a contrast that compares terrestrial to aquatic (e.g. Cats, Dogs, Humans and Birds versus Whales, Fish and Sharks).
- Terrestrial
  - Cats
  - Dogs
  - Humans
  - Birds (Chicken)
- Aquatic
  - Whales (Pilot)
  - Fish (perch)
  - Sharks (Dogfish)

Contrasts *(continued)*

- We compare the mean of 4 things (Cats + Dogs, Humans + Birds)/4 to 3 things (Whales + Fish + Sharks)/3.
- Multipliers are 1/4 and 1/3.

- Terrestrial
  - $^{1}/_{4}$Cats
  - $^{1}/_{4}$Dogs
  - $^{1}/_{4}$Humans
  - $^{1}/_{4}$Birds (Chicken)
- Aquatic
  - $^{1}/_{3}$Whales (Pilot)
  - $^{1}/_{3}$Fish (perch)
  - $^{1}/_{3}$Sharks (Dogfish)

Contrasts *(continued)*

- And if we multiply through by 12 we can get integers.
- This matches 12 means to 12 means.
- One has to be negative so they sum to zero.

- Terrestrial
  - 3Cats
  - 3Dogs
  - 3Humans
  - 3Birds (Chicken)
- Aquatic
  - 4Whales (Pilot)
  - 4Fish (perch)
  - 4Sharks (Dogfish)

Contrasts *(continued)*

- **The multipliers are then**

Tmt Level	multiplier
Cats	3
Dogs	3
Humans	3
Whales	-4
Fish	-4
Sharks	-4
Birds	3

Contrasts *(continued)*

- What if we wanted to compare mammals to others?
- We are still comparing 4 things to 3.
- Make one negative.

- Mammals
  - Cats
  - Dogs
  - Humans
  - Whales
- Others
  - Fish
  - Sharks
  - Birds

Contrasts *(continued)*

- The multipliers are

Tmt Level	multiplier
Cats	3
Dogs	3
Humans	3
Whales	3
Fish	-4
Sharks	-4
Birds	-4

Contrasts *(continued)*

- How about comparing Humans to others? Now we are comparing 1 thing to 6. Give the 6 a coefficient of 1 and the 1 a coefficient of 6.
- One is negative.

- Us
  - Humans
- Them
  - Cats
  - Dogs
  - Whales
  - Fish
  - Sharks
  - Birds

Contrasts *(continued)*

- **The multipliers are**

Tmt Level	multiplier
Cats	1
Dogs	1
Humans	-6
Whales	1
Fish	1
Sharks	1
Birds	1

Contrasts *(continued)*

- One final comparison. How about scaled to unscaled. Comparing 5 things to 2. The two get a 5, the 5 get a 2. One is negative.

- Scaled
  - Fish
  - Sharks
- Unscaled
  - Humans
  - Cats
  - Dogs
  - Whales
  - Birds

Contrasts *(continued)*

- **The multipliers are**

Tmt Level	multiplier
Cats	2
Dogs	2
Humans	2
Whales	2
Fish	-5
Sharks	-5
Birds	2

Contrasts *(continued)*

- **Our contrasts**

Tmt Level	Ter/Aq	Mam	Hum	Scale
Cats	3	3	1	2
Dogs	3	3	1	2
Humans	3	3	-6	2
Whales	-4	3	1	2
Fish	-4	-4	1	-5
Sharks	-4	-4	1	-5
Birds	3	-4	1	2

Contrasts *(continued)*

- Y_i = weekly mean pounds of laundry
- Treatment levels: His, Hers, Ours
- Contrast His to Hers (leave out ours) and Contrast His and Hers to Ours.
- H_0: $\mu_{His} = \mu_{Hers}$, or $\mu_{His} - \mu_{Hers} = 0$
- H_0: ${}^1/_2\mu_{His} + {}^1/_2\mu_{Hers} - \mu_{Ours} = 0$

Tmt Level	His v Hers	H&H v Ours
His		
Hers		
Ours		

Contrasts *(continued)*

- Contrast His to Hers (leave out ours) and Contrast His and Hers to Ours.
- H_0: $\mu_{His} = \mu_{Hers}$, or $\mu_{His} - \mu_{Hers} = 0$
- H_0: $^1/_2\mu_{His} + {}^1/_2\mu_{Hers} - \mu_{Ours} = 0$
- Note: use multiplier 0 to omit a mean.

Tmt Level	His v Hers	H&H v Ours
His	1	1
Hers	-1	1
Ours	0	-2

Contrasts *(continued)*

- We have discussed the t test of contrasts. These can be done either as t tests of F tests (where $t^2 = F$).
- However, some "joint" contrasts may take more than 1 d.f., and these must be done as F tests.
- SAS will use F tests since these can do either.

Contrasts *(continued)*

- **For the fungicide example of a block design, suppose we had certain test we wanted to perform.**
  - **First, compare the CHECK treatment to the others.**
  - **Then compare among selected fungicides one or two against the others.**

Contrasts *(continued)*

- **Contrasts in SAS require only the multipliers for calculation.**
- **Made-up contrast statements for the fungicide example.**

```
PROC MIXED DATA=SOYBEAN  cl COVTEST; CLASSES TREATMNT BLOCK;
  TITLE3 'ANOVA with PROC MIXED - RBD without reps';
  MODEL FAILURES = TREATMNT / htype=3 DDFM=Satterthwaite
                              outp=ResidDataP;
  RANDOM BLOCK;
*** Treatment levels ---------- ARASAN CHECK FERMATE SEMESAN SPERGON;
CONTRAST 'Check v others' TREATMNT    1    -4      1       1      1;
CONTRAST 'S v others'     TREATMNT   -1     0     -1       1      1;
CONTRAST 'A v others'     TREATMNT   -3     0      1       1      1;
CONTRAST 'F v others'     TREATMNT    1     0     -3       1      1;
RUN;
```

Contrasts *(continued)*

- **Contrasts results. Note that each test has an α% chance of error.**

Type 3 Tests of Fixed Effects

Effect	Num DF	Den DF	F Value	Pr > F
TREATMNT	4	16	3.87	0.0219

Contrasts

Label	Num DF	Den DF	F Value	Pr > F
Check v others	1	16	12.43	0.0028
S v others	1	16	1.81	0.1971
A v others	1	16	0.31	0.5865
F v others	1	16	1.00	0.3326

Contrast example

- **For the wire worm fumigant example, suppose we wanted to test for the effect of fumigants (on the average) against the control, and we wanted to test for a difference between the two fumigants. No other tests were of interest.**

Contrast example *(continued)*

- The SAS statements are

```
PROC mixed DATA=FUMIGANT cl; CLASSES FUMIGANT BLOCK REP;
    TITLE3 'ANOVA with PROC MIXED - RBD with reps';
  MODEL WORMS = FUMIGANT / htype=3 DDFM=Satterthwaite
                  outp=ResidDataP outpM=ResidDataPM;
  RANDOM BLOCK FUMIGANT*BLOCK;
*** FUMIGANT levels ----------------   0   C   S;
CONTRAST 'Control v others' FUMIGANT   -2   1   1;
CONTRAST 'C v S'           FUMIGANT    0  -1   1;
RUN;
```

Contrast example *(continued)*

- **Note that in all tests with this example (including other post-ANOVA an error term must be specified). This is not necessary in PROC MIXED.**
-

Contrast example *(continued)*

- **The contrast results are**
  - Type 3 Tests of Fixed Effects

Effect	Num DF	Den DF	F Value	Pr > F
FUMIGANT	2	8	5.98	0.0258

  - Contrasts

Label	Num DF	Den DF	F Value	Pr > F
Control v others	1	8	11.88	0.0087
C v S	1	8	0.08	0.7812

- **The interpretation is clear; the mean of the two fumigants is different from the control, and the two fumigants are not significantly different from each other.**

Orthogonality

- **Two things I mentioned about contrasts.**
  - **They must sum to zero**
  - **They test a contrast of interest to the investigator, so there is not necessarily right or wrong contrast (as long as they test what you intend).**

Orthogonality *(continued)*

- There is one other feature of contrasts that is interesting. When you do a contrast you basically factor off a part of a treatment SS and test that part. If you have 5 d.f. in the treatment, and you do 5 contrasts, will they add up to more, less or equal to the treatment SS?
- It depends. Like TYPE III SS (which contrasts are) they can add up to more or less.

Orthogonality *(continued)*

- **However, if the contrast are orthogonal they will add up to exactly equal the treatment SS.**
- **Orthogonality exists when all pairwise crossproducts sum to zero.**

Level	His-Hers	H&H-Ours	Cross-product
His	1	1	1
Hers	-1	1	-1
Ours	0	-2	0

Orthogonality *(continued)*

- If there are 3 contrasts you must calculate all 3 pairwise crossproducts, and if any one does not sum to zero, the contrasts together are not orthogonal.
- For 4 contrasts, there are 6 pairwise crossproducts, etc.

Orthogonal Polynomial Contrasts

- In cases where treatment levels are quantitative, the test of a linear or quadratic trend may be of interest.
- Tables of contrast multipliers that test polynomial trends are available for equally spaced treatments.

Orthogonal Polynomial Contrasts *(continued)*

- **It is feasible to get multipliers for treatment levels that are NOT equally spaced, but there are an infinite number of such treatments and tables are not available. Multipliers can be obtained from a SAS IML function called ORPOL.**

Orthogonal Polynomial Contrasts *(continued)*

- **Note that these sets of multipliers that do polynomial trends are orthogonal, so they can be placed in any order. In a balanced design they should not change regardless of the order used.**

Orthogonal Polynomial Contrasts *(continued)*

- The tables are available in many statistics textbooks (notes section 23f).
- Typical contrasts are given below.
  - For 3 treatments levels
    - linear -1 0 1
    - quadratic 1 -2 1
  - For 4 treatments levels
    - linear -3 -1 1 3
    - quadratic 1 -1 -1 1
    - cubic -1 3 -3 1

Orthogonal Polynomial Contrasts *(continued)*

- **Additional sets of multipliers are available in the handout up to 9 X_i levels.**
- **When fitted, these one d.f. contrasts are interpreted very much like slopes on a polynomial regression (except TYPE I SS are not needed since they are orthogonal).**
-

Orthogonal Polynomial Contrasts *(continued)*

- **Example: test Millet yield example for quantitative trends. Treatment was row spacing of 2, 4, 6, 8 and 10 inches.**

```
PROC MIXED DATA=MILLET cl; CLASSES ROW COLUMN Spacing;
  TITLE3 'ANOVA with PROC MIXED - Latin Square';
  MODEL YIELD = Spacing / htype=3 DDFM=Satterthwaite
                          outp=ResidDataP;
  RANDOM ROW COLUMN;
*** Row spacing levels -------   A   B   C   D   E;
CONTRAST 'Linear   '   Spacing  -2  -1   0   1   2;
CONTRAST 'Quadratic'   Spacing   2  -1  -2  -1   2;
CONTRAST 'Cubic    '   Spacing  -1   2   0  -2   1;
CONTRAST 'Quartic  '   Spacing   1  -4   6  -4   1;
RUN;
```

Orthogonal Polynomial Contrasts *(continued)*

- Millet yield ANOVA.

Type 3 Tests of Fixed Effects

Effect	Num DF	Den DF	F Value	Pr > F
Spacing	4	12	0.98	0.4523

Contrasts

Label	Num DF	Den DF	F Value	Pr > F
Linear	1	12	3.75	0.0766
Quadratic	1	12	0.03	0.8713
Cubic	1	12	0.14	0.7178
Quartic	1	12	0.02	0.8860

Orthogonal Polynomial Contrasts *(continued)*

- Millet yield ANOVA done in GLM.

Source	DF	Type III SS	Mean Square	F Value	Pr > F
ROW	4	13601.36000	3400.34000	3.22	0.0516
COLUMN	4	6146.16000	1536.54000	1.46	0.2758
TREATMNT	4	4156.56000	1039.14000	0.98	0.4523

Contrast	DF	Contrast SS	Mean Square	F Value	Pr > F
Linear	1	3960.500000	3960.500000	3.75	0.0766
Quadratic	1	28.928571	28.928571	0.03	0.8713
Cubic	1	144.500000	144.500000	0.14	0.7178
Quintic	1	22.631429	22.631429	0.02	0.8860

- Note that contrasts sum to the treatment SS.

Summary

- *A priori* **contrasts are usually preferred because they use fewer d.f. than other post-hoc tests and address research hypotheses directly.**
- **Contrasts are linear combinations of the treatment level means.**
- **Single degree of freedom tests could be done as t-tests. Multiple degree of freedom tests are possible, but use the F test.**

Summary *(continued)*

- Orthognonal tests will sum to the treatment sum of squares. Non-orthogonal contrasts will not, but this is not a problem as long as they test the hypotheses of interest.
- Quantitative treatments should be addressed with orthogonal polynomial contrasts.