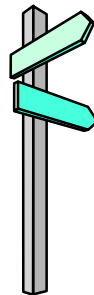# Statistical Techniques II
EXST7015

Expected Mean Squares (EMS)

Rules for creating EMS

- **Expected Mean Squares (EMS) for the Completely Random Design**
  - ► **1) Initially treat all effects as if they were random. Represent each source with a σ^2, and an appropriate subscript. In the simplest case (a treatment and an error term);**

Source	Error	Treatment
Levels	n	t
Var component	σ^2	$\sigma^2 \tau$

Rules for creating EMS *(continued)*

- **2) The lowest level source is generally represented as σ^2. Working up from the lowest level, each source contains the representation for itself and all lower sources.**

Rules for creating EMS *(continued)*

■ **3) The coefficients for each component is the product (if balanced, otherwise see formulas) of the number of observations within the cell for each source level. This will be the product of all the subscripts which occur at lower levels.**

▶ **SOURCE**   **d.f.**   **EMS (random effects)**

▶ **Treatments**   **t-1** $\sigma^2 + n\sigma^2_\tau$

▶ **Error**   **t(n-1)** σ^2

Rules for creating EMS *(continued)*

- **4) FIXED effects; any component which is fixed is represented as a sum of squared effects, and is deleted from all expectations except its own.**

- **EXAMPLE ANOVA table**

  - **SOURCE   d.f.   EMS (fixed effects)**

  - **Treatments   t-1 $\sigma^2 + n\Sigma\tau^2_i/(t-1)$**

  - **Error   t(n-1) σ^2**

- **NOTE that the F test is the same, in this case, for fixed and random effects.**

Rules for creating EMS *(continued)*

- **For a design with more levels**
- **e.g. Ponds, Sites(Ponds), Samples(Sites Ponds), Determinations(Samples Sites Ponds).**
- **MODEL: $Y_{ijkl} = \mu + \tau_i + \delta_{ij} + \gamma_{ijk} + \varepsilon_{ijkl}$**
- **let Ponds = t = 5; Sites = s = 7; Samples = b = 3; Det = d = 4**
- **This is the typical situation for a hierarchical design (fully nested)**

Rules for creating EMS *(continued)*

- **1) Initially treat all effects as if random. Represent each source with a s2 and an appropriate subscript. In this case;**
  - ►**Source   Tmt   ExErr   SampErr SubSamp**
  - ►**Levels   t   s   b   d**
  - ►**Var comp** $\sigma^2\tau$ $\sigma^2\delta$ $\sigma^2\gamma$ σ^2
- **2) Working up from the lowest level, set up each source to contain itself and all lower sources.**

Rules for creating EMS *(continued)*

- **3) This design is balanced, so the coefficient for each component is the product of the number of observations within the cell (the product of all the subscripts which occur at lower levels).**

Rules for creating EMS *(continued)*

- **Expected mean squares**
- **SOURCE   d.f.   EMS coefficient**
- **Ponds   t-1 $\sigma^2 + d\sigma^2_\gamma + db\sigma^2_\delta + dbs\sigma^2_\tau$**
- **Sites(P)   t(s-1) $\sigma^2 + d\sigma^2_\gamma + db\sigma^2_\delta$**
- **Samp(P S) ts(b-1) $\sigma^2 + d\sigma^2_\gamma$**
- **Error   tsb(d-1) σ^2**
- **Total   tsbd - 1**

Rules for creating EMS *(continued)*

- **Expected mean squares with numerical coefficients**

SOURCE	d.f.	EMS coefficient
Ponds	t-1	$\sigma^2 + 4\sigma^2_\gamma + 12\sigma^2_\delta + 84\sigma^2_\tau$
Sites(P)	t(s-1)	$\sigma^2 + 4\sigma^2_\gamma + 12\sigma^2_\delta$
Samp(P S)	ts(b-1)	$\sigma^2 + 4\sigma^2_\gamma$
Error	tsb(d-1)	σ^2
Total	tsbd - 1	

Rules for creating EMS *(continued)*

■ **For FIXED effects, any effect which is fixed is deleted from all expectations except its own. In our CRD example it is unlikely that any source except treatments would be fixed, since the others are sources of random error. Therefore, if the treatments were fixed we would only change dbsσ^2_τ to dbs$\Sigma\tau^2$/(t-1), or numerically $84\sigma^2_\tau$ becomes $84\Sigma\tau^2_i$/4 or $21\Sigma\tau^2_i$.**

Rules for creating EMS *(continued)*

- **Block designs proceed the same except that the blocks are not nested and have their own source line just like treatments. The interaction effect has (b-1)(t-1) d.f.**

- **We will let SAS get most of our EMS.**

- **However, you should be able to get EMS for simple designs and you should be able to examine SAS produced EMS and determine which sources are tested with which error terms.**

Rules for creating EMS *(continued)*

- **There is one other rule that SAS uses that will be relevant to treatment arrangements when we get to those.**
  - ▶**EMS for interactions of random effects are random.**
  - ▶**EMS for interactions of fixed effects are fixed.**
  - ▶**Interactions with mixed effects are random. If there is any random source in an interaction, that interaction EMS is a random effect.**

EMS, what good are they?

- **Once upon a time you had to be able to determine the EMS to figure out what terms got tested with what error terms.**

- **Then life got easy when SAS began providing the EMS in PROC GLM, but you still had to specify which terms got tested with which other terms.**

EMS, what good are they? *(continued)*

- **Then along comes PROC MIXED.**
- **PROC MIXED not longer estimates the EMS, it estimates the individual variance components of the EMS, but not the EMS themselves.**
- **It will also determine which terms should be used to test other terms.**
- **And life gets even easier.**

EMS, what good are they? *(continued)*

- **But, you will notice in some more complicated Experimental Designs that the different sources of variation do not all have the same degrees of freedom for the error.**

- **We therefore look at the EMS in order to understand why.**

EMS, what good are they? *(continued)*

- PROC MIXED can also produce the EMS, but in order to get them you must force PROC MIXED to do things the "old fashion way".

- So you will get the EMS, but will not get most of the other features of the more modern analysis.

- This is done by specifying a