The Z transformation of any normal distribution

Most practical applications of the Z distribution will require that we take a real distribution $\{N(\mu,\sigma^2)\}$ and convert it to a Z distribution $\{N(0,1)\}$, calculate some probability statement, and then convert the results back to the original distribution.

So we work with two distributions

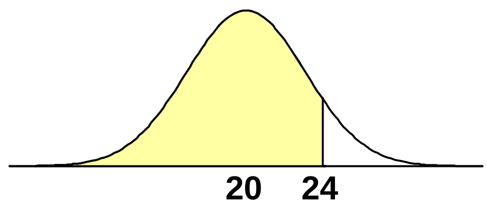
$Y \sim N(\mu,\sigma^2)$

$Z \sim N(0,1)$

In order to convert the observed Y distribution to the more workable Z distribution we need to transform the distribution. We use the Z transformation.

$Z_i = (Y_i - \mu) / \sigma$

For example, suppose we have a distribution where $Y \sim N(20,16)$ and we wish to determine $P(Y \le 24)$. We convert the probability statement for the original distribution into a Z distribution using our transformation.



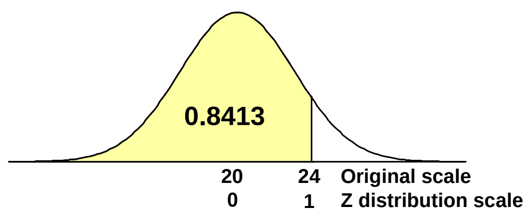
Transformation: $Z_i = (Y_i - \mu) / \sigma$ for N(20, 16)

$$P(Y \le 24) = P\left(\frac{(Y_i - \mu)}{\sigma} \le \frac{(24-20)}{4}\right)$$

$$= P(Z \le 1)$$

$P(Z \le 1) = 1 - P(Z \ge 1) = 1 - 0.1587 = 0.8413$

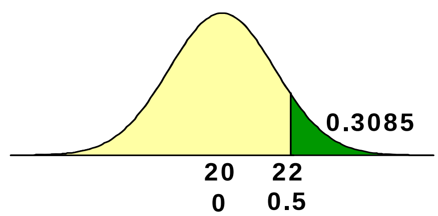
So, $P(Y \le 24) = 0.8413$



Another example

Using the same distribution $Y \sim N(20,16)$ where we wish to determine $P(Y \ge 22)$.

$$P(Y \ge 22) = P\left(\frac{(Y_i - \mu)}{\sigma} \ge \frac{(22-20)}{4}\right) = P(Z \ge 0.5)$$



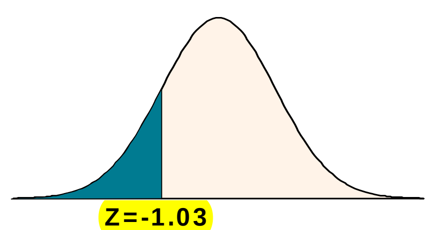
From the table we determine that $P(Z \ge 0.5) = 0.3085$

Another type of example

Find Y_0, where $P(Y \le Y_0) = 0.1515$ for the same distribution, $Y \sim N(20, 16)$. Again, using our transformation,

$$P(Y \le Y_0) = 0.1515 = P\left(\frac{(Y_i - \mu)}{\sigma} \le \frac{(Y_0 - \mu)}{\sigma}\right) =$$

$P(Z \le Z_0) = 0.1515$



Notice that the sign is $\le$ and the probability small (less than 0.5), so we are in the lower half of the distribution. From the table the Z value is 1.03, so $Z_0 = -1.03$. So we know that $Z_0 = -1.03$, an area in the lower half of the distribution, but we don't know the value of Y_0 yet and our original problem was to find $P(Y \le Y_0) = 0.1515$. So we need to transform back to the Y scale. This back-transformation is a reversal of the Z transformation.