Using the SAS **PROC TTEST;**

1) An F test is provided to test H_0: $\sigma_1^2 = \sigma_2^2$. The F is calculated as $F = \frac{\text{larger of } S_1^2,\ S_2^2}{\text{smaller of } S_1^2,\ S_2^2}$. This test should be examined first to decide which of the following two cases is appropriate.

2a) For use in the event that the variances are equal, PROC TTEST calculates the t-test sample statistic as $t = \frac{\bar{d} - \delta}{S_{\bar{d}}} = \frac{(\bar{Y}_1 - \bar{Y}_2) - (\mu_1 - \mu_2)}{\sqrt{S_p^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$.

The pooled variance is a weighted mean of the two individual variances, where the weights are the degrees of freedom. The pooled variance is given by the equation $S_p^2 = \frac{SS_1 + SS_2}{\gamma_1 + \gamma_2} = \frac{SS_1 + SS_2}{(n_1 - 1) + (n_2 - 1)} = \frac{(n_1 - 1)S_1^2 + (n_2 - 1)S_2^2}{(n_1 - 1) + (n_2 - 1)}$. The degrees of freedom for this variance are $(n_1\text{-}1) + (n_2\text{-}1)$.

2b) For use in the event that the variances are unequal, PROC TTEST calculates a second t-test statistic. The second t-test sample test statistic is calculated as

$$t = \frac{\bar{d} - \delta}{S_{\bar{d}}} = \frac{(\bar{Y}_1 - \bar{Y}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}}$$

The calculations for the Prob (>t) for this case ($\sigma_1^2 \neq \sigma_2^2$) is more complicated since the value of df is in doubt. We can calculate a weighted degrees of freedom. This value will be, at a minimum, equal to the smaller of either (n_1-1) or (n_2-1). If we use Satterthwaite's approximation it will range from this minimum up to ($n_1 + n_2$ - 2), depending on the disparity in the variances. Satterthwaite's approximation is given as

$$\text{d.f.} = \gamma \approx \frac{\left(\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}\right)^2}{\frac{\left(S_1^2/n_1\right)^2}{n_1 - 1} + \frac{\left(S_2^2/n_2\right)^2}{n_2 - 1}}$$