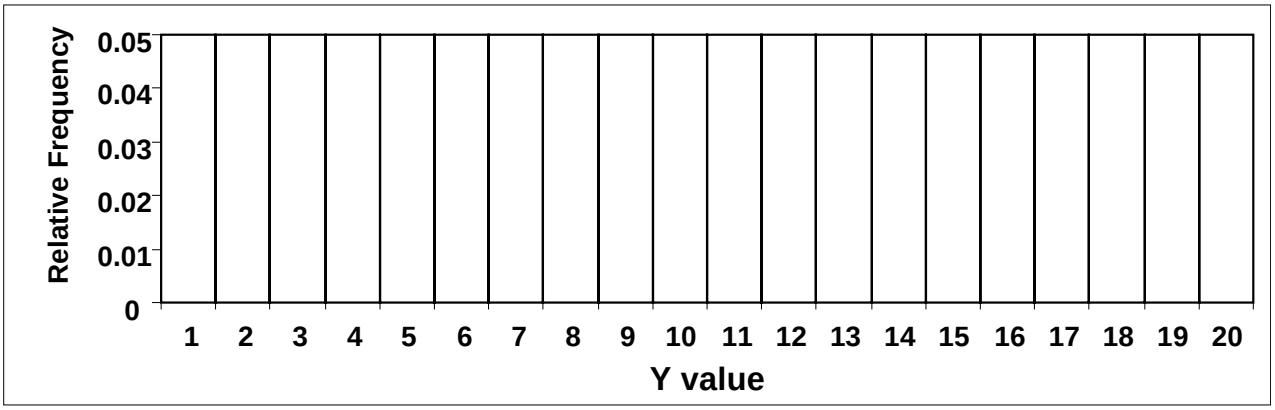
EXPERIMENTAL STATISTICS 7005 Name _____
February 18, 1999 Exam 1 All sections : Geaghan

Read Carefully. Give an answer in the form of a number or numeric expression where possible. Show all calculations for possible partial credit. Us a value of 0.05 for α if not specified. Z tables and t-tables are provided separately. You may keep these tables.



1) 8 points – Examine the discrete uniform distribution U(1,20) above. Calculate the probabilities requested below.

a) $P(Y > 10)$ = _____

b) $P(Y = 3)$ = _____

c) $P(4 \le Y \le 16)$ = _____

d) $P(Y \ge 2)$ = _____

2) 3 points — Circle the answers below that are needed as assumptions for a Z test of hypothesis.

a) Normality

b) Mean = 0

c) Known variance

d) Independence

e) Large sample size

3) 3 points – Which of the following is true of the mean, median and mode for data with a positive skew?

a) Mean < Mode < Median

b) Mode < Median < Mean

c) Mean < Median < Mode

d) Median < Mode < Mean

4) 14 points – Answer the following questions as True or False by circling the appropriate letter.

T F a) Establishing an hypothesis is part of the scientific method.

T F b) The assumption of normality is more easily met with larger sample sizes according to the Central Limit Theorem.

T F c) For the Normal Distribution, it is true that the Mode, Median and Mean are equal.

T F d) Deviations of observations (i.e. $(Y_i - \overline{Y})$) from the sample mean always sum to zero.

T F e) When data is "corrected" with a correction factor, it is corrected for random sampling variation.

T F f) If data from a sample has been transformed by adding 100 no detransformation is needed for the variance.

T F g) If a sample of data has the units "millimeters" for the mean, the variance will have units of "millimeters squared".

5) 3 points – If you cut an honest deck of cards (with 54 different cards with equally likely values of A, 2, 3, 4, 5, 6, 7, 8, 9, 10, J, Q and K and one of each equally likely suit for each value (e.g. ♣, ♦, ♥ and ♠)) plus <u>two Jokers</u>, the probability of getting a Joker is equal to

_____ .

6) 12 points – An investigator is studying a population of Holly trees from an island off North Carolina. There are 100 trees on the island with a mean diameter of 15 inches and a standard deviation of 5 inches. Assuming that the tree diameters follow a normal distribution, answer the questions below pertaining to this population.

a) What is the probability that an individual selected at random would have a diameter greater than 23 inches?

b) What is the probability that the mean diameter of 6 individuals selected at random would be less than 9 inches?

c) What is the probability that an individual selected at random would have a diameter between 10 and 25 inches?

7) 10 points — Find the probabilities indicated below.

a) $P(Z \leq 0.81) =$ P value = _____

b) $P(Z \geq -1.27) =$ _____ P value = _____

c) $P(|Z| \leq 1.22) =$ _____ P value = _____

d) $P(|Z| \geq 2.05) =$ _____ P value = _____

e) $P(-1.11 \leq Z \leq 1.35) =$_____ P value = _____

8) 16 points — Find the value of Z0 which yields the following probabilities.

a) $P(Z \leq Z_0) = 0.1056$ $Z_0 =$ _____

b) $P(Z \geq Z_0) = 0.0116$ $Z_0 =$ _____

c) $P(|Z| \leq Z_0) = 0.2420$ $Z_0 =$ _____

d) $P(Z_0 \leq Z) = 0.6469$ $Z_0 =$ _____

e) $P(|Z| \geq Z_0) = 0.2040$ $Z_0 =$ _____

f) $P(Z_0 \geq Z) = 0.1170$ $Z_0 =$ _____

g) $P(-Z_0 \leq Z \leq +Z_0) = 0.2960$ $Z_0 =$ _____

h) $P(Z_0 \leq Z \leq 1.00) = 0.6500$ $Z_0 =$ _____

9) 12 points — Suppose that a population under study is known to follow a normal distribution with the following characteristics. $\mu = 100$, $\sigma^2 = 100$, $\sigma = 10$ and where (or if) applicable, n = 25. Find the probability (P value) or value of Y0 for the following statements.

a) $P(85 \leq Y \leq 115) =$ P value = _____

b) $P(Y_0 \leq Y \leq 110) = 0.8300$ $Y_0 =$ _____

c) $P(Y \leq Y_0) = 0.9370$ $Y_0 =$ _____

d) $P(97 \leq \bar{Y} \leq 105) =$ P value = _____

11) Suppose that the data producing the SAS output provided is considered to be a POPULATION of unique individuals. The population was created by averaging 10 numbers from a uniform distribution (Uniform(0,1)) and multiplying by a constant and adding another constant to create a distribution with a mean of 50 and a variance of 20. It was thought that averaging 10 values from a Uniform distribution would create a Normal distribution.

Answer the questions below about the population. BE VERY CAREFUL ABOUT ANSWERING THESE QUESTIONS BECAUSE PROC UNIVARIATE DOES CALCULATIONS FOR SAMPLES, NOT POPULATIONS!

a) 5 points – What is the actual VARIANCE and standard deviation for this population? Give at least two decimal place precision! $\sigma^2 =$ _____ , $\sigma =$ _____

b) 6 points – What is the probability that an individual drawn from this population will have a value between 52 and 56?.

$$P(52 \leq Y \leq 56) =$$ _____

c) 8 points – Do a test of hypothesis (7 steps) to test the hypothesis that the mean of this population is equal to the hypothesized value of 50.

Examine the output below and answer the questions on the last page of the exam.

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EXST7005 Exam 1 problem
Univariate summary statistics

Univariate Procedure
Variable=YVALUE

                Moments
N                100  Sum Wgts        100
Mean          49.4813  Sum          4948.13
Std Dev      3.777262  Variance    14.26771
Skewness     -0.15589  Kurtosis     -0.4252
USS          246252.4  CSS          1412.503
CV           7.633717  Std Mean     0.377726
T:Mean=0     130.9978  Pr>|T|         0.0001
Num ^= 0          100  Num > 0           100
M(Sign)            50  Pr>=|M|        0.0001
Sgn Rank         2525  Pr>=|S|        0.0001
W:Normal     0.979813  Pr<W           0.5173

            Quantiles(Def=5)
100% Max       57.46       99%      57.015
 75% Q3         52.4       95%       55.49
 50% Med       49.37       90%        54.5
 25% Q1        46.93       10%      44.645
  0% Min       39.19        5%       42.76
                            1%       40.75
Range          18.27
Q3-Q1           5.47
Mode           39.19

                Extremes
   Lowest    Obs     Highest    Obs
    39.19(      95)    55.66(      57)
    42.31(       7)    55.71(      23)
    42.65(      33)    56.21(      14)
    42.73(      79)    56.57(      39)
    42.74(      80)    57.46(      13)

  Stem Leaf                     #  Boxplot
    57 5                        1     |
    56 26                       2     |
    55 1377                     4     |
    54 0223799                  7     |
    53 114468                   6     |
    52 01355778                 8  +-----+
    51 0112344668              10  |     |
    50 2445569                  7  |     |
    49 2223345689              10  *--+--*
    48 222234568                9  |     |
    47 02344667789             11  |     |
    46 11233568                 8  +-----+
    45 1334568                  7     |
    44 2                        1     |
    43 126                      3     |
    42 36778                    5     |
    41                                |
    40                                |
    39 2                        1     |
       ----+----+----+----+
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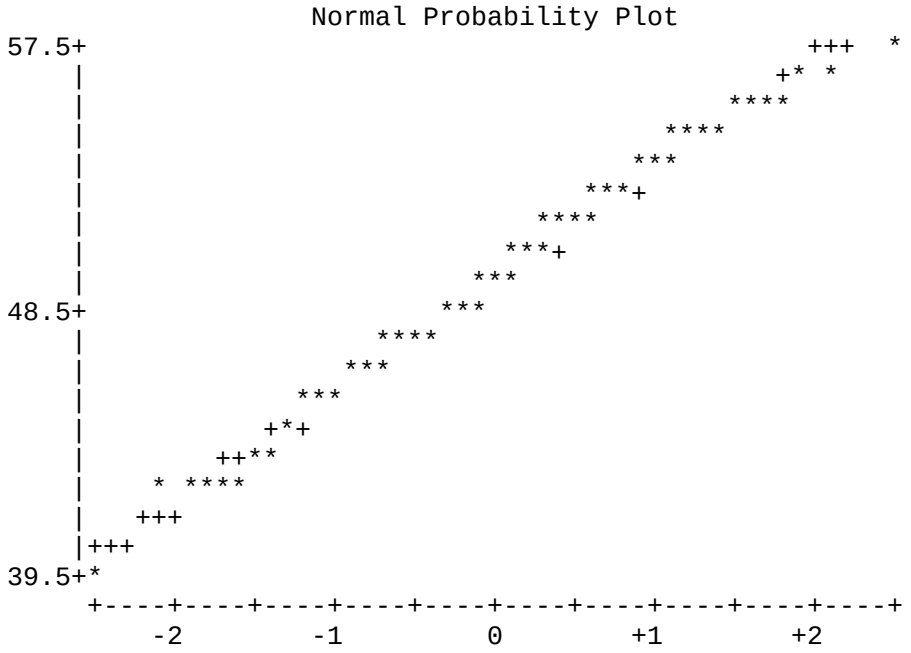

EXST7005 Exam 1 problem

Histogram

Frequency