Chapter 9 : Multiple Regression

The first example of multiple regression is a designed experiment. The experiment involves the development of flowers on “Meadowfoam” a small cultivated plant used for its seed oil. The data for this analysis comes from one experiment on this plant that examined flower production. There were two treatments in this experiment. The first was 6 levels of light intensity (150, 300, 450, 600, 750 and 900 μmol/m^2/sec) and the second was the timing of the application of light, either early or late in the flower growing period.

```
1          *******************************************************;
2          *** The effect of light on Meadowfoam flowering.      ***;
3          *** Results of an experiment where the effedt of six  ***;
4          *** levels of light intensity and the timing of the   ***;
5          *** light treatment was investigated.                 ***;
6          *******************************************************;
7
8          dm'log;clear;output;clear';
9          options nodate nocenter nonumber ps=512 ls=99 nolabel;
10         ODS HTML style=minimal rs=none
10       ! body='C:\Geaghan\Current\EXST3201\Fall2005\SAS\Meadowfoam01.html' ;
NOTE: Writing HTML Body file:
   C:\Geaghan\Current\EXST3201\Fall2005\SAS\Meadowfoam01.html
11
12         Title1 'Chapter 9 : The effect of light on Meadowfoam flowering';
13         filename input1
   'C:\Geaghan\Current\EXST3201\Datasets\ASCII\case0901.csv';
14
15         data Meadowfoam; infile input1 missover DSD dlm="," firstobs=2;
16            input FLOWERS TIME INTENSity;
17              label Flowers = 'Average number of flowers per plant'
18                    Time = 'Early and Late'
19                    Intensity = 'Level of light intensity';
20              Time0 = Time - 1;
21           TimeName = 'Early'; if time eq 1 then Timename = 'Late';
22         datalines;
NOTE: The infile INPUT1 is:
      File Name=C:\Geaghan\Current\EXST3201\Datasets\ASCII\case0901.csv,
      RECFM=V,LRECL=256
NOTE: 24 records were read from the infile INPUT1.
      The minimum record length was 8.
      The maximum record length was 24.
NOTE: The data set WORK.MEADOWFOAM has 24 observations and 5 variables.
NOTE: DATA statement used (Total process time):
      real time           0.02 seconds
      cpu time            0.02 seconds
23         run;
24
25         PROC PRINT DATA=Meadowfoam; TITLE2 'Raw data Listing'; RUN;
NOTE: There were 24 observations read from the data set WORK.MEADOWFOAM.
NOTE: The PROCEDURE PRINT printed page 1.
NOTE: PROCEDURE PRINT used (Total process time):
      real time           0.11 seconds
      cpu time            0.02 seconds
26
```

I modified the data so that, in addition to the variables “FLOWERS, TIME AND INTENSITY” the variable time which originally had values of (1, 2) was also expressed as (0, 1) and as (Early, Late).

```
Chapter 9 : The effect of light on Meadowfoam flowering
Raw data Listing
```

Obs	FLOWERS	TIME	INTENSity	Time0	Time Name
1	62.3000	1	150	0	Late
2	77.4000	1	150	0	Late
3	55.3000	1	300	0	Late
4	54.2000	1	300	0	Late
5	49.6000	1	450	0	Late
6	61.9000	1	450	0	Late
7	39.4000	1	600	0	Late
8	45.7000	1	600	0	Late
9	31.3000	1	750	0	Late
10	44.9000	1	750	0	Late
11	36.8000	1	900	0	Late
12	41.9000	1	900	0	Late
13	77.8000	2	150	1	Early
14	75.6000	2	150	1	Early
15	69.1000	2	300	1	Early
16	78.0000	2	300	1	Early
17	57.0000	2	450	1	Early
18	71.1000	2	450	1	Early
19	62.9000	2	600	1	Early
20	52.2000	2	600	1	Early
21	60.3000	2	750	1	Early
22	45.6000	2	750	1	Early
23	52.6000	2	900	1	Early
24	44.4000	2	900	1	Early

```
27         options ps=52 ls=111;
28         proc plot data=Meadowfoam;  TITLE2 'Plot of the raw data';
29            plot Flowers * Intensity = TimeName;
30         RUN;
30       !      OPTIONS PS=256;
31
```

```
Chapter 9 : The effect of light on Meadowfoam flowering
Plot of the raw data

                     Plot of FLOWERS*INTENSity.  Symbol is value of TimeName.
  FLOWERS |
          |
       80 +
          |  E              E
          |  L
          |  E
          |
          |                                E
       70 +
          |                 E
          |
          |
          |                                               E
          |  L                             L
       60 +                                                              E
          |
          |                                E
          |                 L
          |                                                                             E
          |                                               E
       50 +                                L
          |
          |
          |                                               L              L              E
          |
          |                                                                             L
       40 +                                               L
          |
          |                                                                             L
          |
          |
          |                                                              L
       30 +
          |
          ---+--------------+--------------+--------------+--------------+--------------+-
            150            300            450            600            750            900
                                              INTENSity
NOTE: 2 obs hidden.
```

First examine the raw data plot. Note the expression of the first letter from “Early” and “Late”.

The first model was fitted as a SLR to the quantitative variable “TIME”.

```
32         Title2 'Initial fit of the raw data to TIME';
NOTE: There were 24 observations read from the data set WORK.MEADOWFOAM.
NOTE: The PROCEDURE PLOT printed page 2.
NOTE: PROCEDURE PLOT used (Total process time):
      real time           0.06 seconds
      cpu time            0.00 seconds
```

```
33         PROC REG DATA=Meadowfoam lineprinter;
34            MODEL Flowers = time; RUN;
NOTE: The PROCEDURE REG printed page 3.
NOTE: PROCEDURE REG used (Total process time):
      real time           0.06 seconds
      cpu time            0.02 seconds
35
Chapter 9 : The effect of light on Meadowfoam flowering
Initial fit of the raw data to TIME

The REG Procedure
Model: MODEL1
Dependent Variable: FLOWERS

Number of Observations Read          24
Number of Observations Used          24

Analysis of Variance
                                    Sum of           Mean
Source                    DF       Squares         Square    F Value    Pr > F
Model                      1     886.95034      886.95034       5.65    0.0265
Error                     22    3450.98592      156.86300
Corrected Total           23    4337.93627

Root MSE             12.52450    R-Square     0.2045
Dependent Mean       56.13750    Adj R-Sq     0.1683
Coeff Var            22.31039

 Parameter Estimates
                     Parameter       Standard
Variable     DF       Estimate          Error    t Value    Pr > |t|
Intercept     1       37.90000        8.08453       4.69      0.0001
TIME          1       12.15833        5.11310       2.38      0.0265
```

The next model was fitted as a SLR to the quantitative variable "intensity".

```
36         Title2 'Initial fit of the raw data to INTENSITY';
37         PROC REG DATA=Meadowfoam lineprinter;
38            MODEL Flowers = Intensity;
39            output out=next r=resid;
40         RUN;
41
NOTE: The data set WORK.NEXT has 24 observations and 6 variables.
NOTE: The PROCEDURE REG printed page 4.
NOTE: PROCEDURE REG used (Total process time):
      real time           0.10 seconds
      cpu time            0.04 seconds


Chapter 9 : The effect of light on Meadowfoam flowering
Initial fit of the raw data to INTENSITY

The REG Procedure
Model: MODEL1
Dependent Variable: FLOWERS

Number of Observations Read          24
Number of Observations Used          24
```

```
Analysis of Variance               Sum of            Mean
Source                   DF       Squares          Square    F Value    Pr > F
Model                     1    2579.75004      2579.75004      32.28    <.0001
Error                    22    1758.18622        79.91756
Corrected Total          23    4337.93627

Root MSE              8.93966    R-Square     0.5947
Dependent Mean       56.13750    Adj R-Sq     0.5763
Coeff Var            15.92458

Parameter Estimates
                     Parameter       Standard
Variable     DF       Estimate          Error    t Value    Pr > |t|
Intercept     1       77.38500        4.16119      18.60      <.0001
INTENSity     1       -0.04047        0.00712      -5.68      <.0001


42         options ps=52 ls=111;
43         proc plot data=next;  TITLE2 'Plot of the raw data';
44            plot resid * Intensity = TimeName;
45         RUN;
45       !      OPTIONS PS=256;
NOTE: There were 24 observations read from the data set WORK.NEXT.
NOTE: The PROCEDURE PLOT printed page 5.
NOTE: PROCEDURE PLOT used (Total process time):
      real time           0.07 seconds
      cpu time            0.02 seconds
46
```

Note the general separation in the "E" and "L" groups below. The were not included in this model.

```
Chapter 9 : The effect of light on Meadowfoam flowering
Plot of the raw data

                            Plot of resid*INTENSity.  Symbol is value of TimeName.
       resid |
          15 +
             |
             |                   E                                                  E
             |                                    E                                                  E
             |
          10 +                                                     E
             |
             |
             |
             |  L
           5 +
             |  E                E
             |                                    L                                                  E
             |
             |                                                                                       L
           0 +
             |                                                     E                E
             |                                    E                                 L
             |
             |                                                                                       L
          -5 +
             |
             |                                                     L
             |
             |  L
         -10 +                   L                L
             |                   L
             |
             |
             |                                                     L
         -15 +
             |                                                                      L
             |
             |
             |
         -20 +
             |
             ---+----------------+----------------+----------------+----------------+----------------+--
               150              300              450              600              750              900
                                                      INTENSity
NOTE: 1 obs hidden.
```

```
          Title2 'Multiple regression';
          options ps=512 ls=111;
          PROC REG DATA=Meadowfoam lineprinter;
             MODEL Flowers = Intensity time;
             output out=next r=resid p=YHat;
          RUN;
NOTE: The data set WORK.NEXT has 24 observations and 7 variables.
NOTE: The PROCEDURE REG printed page 6.
NOTE: PROCEDURE REG used (Total process time):
      real time           0.14 seconds
      cpu time            0.08 seconds
```

```
Chapter 9 : The effect of light on Meadowfoam flowering
Multiple regression

The REG Procedure
Model: MODEL1
Dependent Variable: FLOWERS

Number of Observations Read          24
Number of Observations Used          24

Analysis of Variance
                                        Sum of           Mean
Source                    DF           Squares         Square    F Value    Pr > F
Model                      2        3466.70039     1733.35019      41.78    <.0001
Error                     21         871.23588       41.48742
Corrected Total           23        4337.93627

Root MSE              6.44107    R-Square     0.7992
Dependent Mean       56.13750    Adj R-Sq     0.7800
Coeff Var            11.47374

Parameter Estimates
                     Parameter       Standard
Variable     DF       Estimate          Error    t Value    Pr > |t|
Intercept     1       59.14750        4.95447      11.94      <.0001
INTENSity     1       -0.04047        0.00513      -7.89      <.0001
TIME          1       12.15833        2.62956       4.62      0.0001
```

Is there an interpretation of the slope and intercept? Can plants grow flowers if light intensity is zero? The units on the slope is “flowers per μmol/m^2/sec of light intensity”

Calculation of Extra Sum of Squares.

SSXT = 886.95034

SSXI = 2579.75004

SSXT | XI = 3466.70039 – 2579.75004 = 886.95034

SSXI | XT = 3466.70039 – 886.95034 = 2579.75004

How come the SS for each variable is not modified by the other???

```
52       !      OPTIONS PS=45;
53          TITLE3 'Plot of residuals';
54          Proc plot; PLOT resid*Intensity=timename / vref=0;
NOTE: There were 24 observations read from the data set WORK.NEXT.
NOTE: The PROCEDURE PLOT printed page 7.
NOTE: PROCEDURE PLOT used (Total process time):
      real time           0.13 seconds
      cpu time            0.03 seconds
Chapter 9 : The effect of light on Meadowfoam flowering
Multiple regression
Plot of residuals
```

Note that there is no longer appreciable separation in the “E” and “L” groups.

```
                       Plot of resid*INTENSity.  Symbol is value of TimeName.
    resid |
       15 +
          |
          |
          |  L
          |
          |
       10 +
          |                                  L
          |
          |                                                                  E
          |                  E                                                               L
          |                                  E                                               E
        5 +
          |                                                                  L
          |                                                  E
          |
          |                                                                                  L
          |
        0 +--E----------------------------------------------------------------------------------
          |
          |  E                                               L
          |                  E                                                               E
          |  L                               L
          |                  L
       -5 +                  L
          |
          |                                                  E
          |                                                  L               E
          |                                  E
          |
      -10 +                                                                  L
          |
          ---+---------------+---------------+---------------+---------------+---------------+--
            150             300             450             600             750             900
                                                INTENSity
```

```
55          Proc plot; PLOT resid*YHat=time / vref=0;
56          RUN;
56       !      options ps=512 ls=111;
57
NOTE: There were 24 observations read from the data set WORK.NEXT.
NOTE: The PROCEDURE PLOT printed page 8.
NOTE: PROCEDURE PLOT used (Total process time):
      real time           0.04 seconds
      cpu time            0.00 seconds
```

Chapter 9 : The effect of light on Meadowfoam flowering
Multiple regression
Plot of residuals

```
                                 Plot of resid*YHat.  Symbol is value of TIME.
resid |
   15 +
      |
      |
      |                                                                     1
      |
      |
   10 +
      |                                          1
      |
      |                                          2
      |  1                                                                               2
      |                            2                                        2
    5 +
      |               1
      |                                                       2
      |
      |  1
      |
    0 +-----------------------------------------------------------------------------------------------2--------
      |
      |                            1                                                                  2
      |                            2                                                     2
      |                                          1                          1
      |                                                       1
   -5 +                                                       1
      |
      |                                                       2
      |                            1             2
      |                                                                     2
      |
  -10 +               1
      |
      ---+----------+----------+----------+----------+----------+----------+----------+----------+----------+--
        35         40         45         50         55         60         65         70         75         80
                                                         YHat
```

```
58         PROC UNIVARIATE DATA=NEXT NORMAL PLOT; VAR resid;
59         RUN;
NOTE: The PROCEDURE UNIVARIATE printed page 9.
NOTE: PROCEDURE UNIVARIATE used (Total process time):
      real time           0.05 seconds
      cpu time            0.02 seconds
60
```

Chapter 9 : The effect of light on Meadowfoam flowering
Multiple regression
Plot of residuals

The UNIVARIATE Procedure
Variable: resid

Moments

N	24	Sum Weights	24
Mean	0	Sum Observations	0
Std Deviation	6.15465847	Variance	37.8798209
Skewness	0.21089332	Kurtosis	-1.0360321
Uncorrected SS	871.23588	Corrected SS	871.23588
Coeff Variation	.	Std Error Mean	1.2563144

Basic Statistical Measures

Location		Variability	
Mean	0.00000	Std Deviation	6.15466
Median	-1.55821	Variance	37.87982
Mode	.	Range	21.81715
		Interquartile Range	10.11845

```
Tests for Location: Mu0=0
Test           -Statistic-    -----p Value------
Student's t    t         0    Pr > |t|    1.0000
Sign           M        -1    Pr >= |M|   0.8388
Signed Rank    S        -2    Pr >= |S|   0.9559

Tests for Normality
Test                  --Statistic---    -----p Value------
Shapiro-Wilk          W     0.955588    Pr < W      0.3563
Kolmogorov-Smirnov    D     0.126766    Pr > D     >0.1500
Cramer-von Mises      W-Sq  0.068129    Pr > W-Sq  >0.2500
Anderson-Darling      A-Sq  0.405333    Pr > A-Sq  >0.2500

Quantiles (Definition 5)
Quantile       Estimate
100% Max       12.16488
99%            12.16488
95%             8.80631
90%             7.18940
75% Q3          5.70405
50% Median     -1.55821
25% Q1         -4.41441
10%            -7.62298
5%             -8.25202
1%             -9.65226
0% Min         -9.65226

Extreme Observations
------Lowest-----        ------Highest-----
   Value      Obs           Value      Obs
-9.65226        9         6.67726       16
-8.25202       17         7.01845       12
-7.62298        7         7.18940       21
-7.51060       22         8.80631        6
-6.98131       20        12.16488        2
```

```
   Stem Leaf  Boxplot                        Normal Probability Plot
     12 2                        1     |          13+                                            *+++
     10                                |            |                                          +++
      8 8                        1     |            |                                      ++*+
      6 702                      3     |            |                                  **+*
      4 68                       2  +-----+         |                               **++
      2 79                       2  |     |         |                             **+
      0 49                       2  |  +  |         |                         ++**
     -0 83                       2  *-----*         |                      ++* *
     -2 95962                    5  |     |         |                   *****
     -4 0                        1  +-----+         |                ++*
     -6 650                      3     |            |             *+**
     -8 73                       2     |          -9+      *  +*++
        ----+----+----+----+                         +----+----+----+----+----+----+----+----+----+----+
                                                         -2        -1         0        +1        +2
```

Other models discussed by the text

Simple linear regression: $\mu_{\{Y|X\}} = \beta_0 + \beta_1 X_i$

Basic multiple linear regression: $\mu_{\{Y|X_1,X_2\}} = \beta_0 + \beta_1 X_{1i} + \beta_2 X_{2i}$

$$\mu_{\{Y|X_1,X_2,X_3,\ldots,X_k\}} = \beta_0 + \beta_1 X_{1i} + \beta_2 X_{2i} + \beta_3 X_{3i} + \ldots + \beta_k X_{ki}$$

Polynomial regression: $\mu_{\{Y|X,X^2\}} = \beta_0 + \beta_1 X_i + \beta_2 X_i^2$

$$\mu_{\{Y|X,X^2X^3\ldots X^k\}} = \beta_0 + \beta_1 X_i + \beta_2 X_i^2 + \beta_3 X_i^3 + \ldots + \beta_k X_i^k$$

Multiple regression with interaction: $\mu_{\{Y|X_1,X_2,X_1X_2\}} = \beta_0 + \beta_1 X_{1i} + \beta_2 X_{2i} + \beta_3 X_{1i} X_{2i}$

Multiple regression with transformation: $\mu_{\{Y|\log(X_1),\log(X_2)\}} = \beta_0 + \beta_1 \log(X_{1i}) + \beta_2 \log(X_{2i})$

Analysis of covariance is a least squares model that has a mix of quantitative variables (typical regression variables) and indicator variables (binary variables coded as 0 or 1). The models fitted are as follows:

Simple linear regression: $\hat{Y}_i = b_0 + b_1 X_{1i}$

Basic multiple linear regression: $\hat{Y}_i = b_0 + b_1 X_{1i} + b_2 X_{2i}$

When group = 0: $\hat{Y}_i = b_0 + b_1 X_{1i} + b_2 X_{2i} = b_0 + b_1 X_{1i} + b_2 0 = b_0 + b_1 X_{1i}$

When group = 1: $\hat{Y}_i = b_0 + b_1 X_{1i} + b_2 X_{2i} = b_0 + b_1 X_{1i} + b_2 1 = (b_0 + b_2) + b_1 X_{1i}$

multiple linear regression with interaction: $\hat{Y}_i = b_0 + b_1 X_{1i} + b_2 X_{2i} + b_3 X_{1i} X_{2i}$

When group = 0: $\hat{Y}_i = b_0 + b_1 X_{1i} + b_2 X_{2i} + b_3 X_{1i} X_{2i} = b_0 + b_1 X_{1i} + b_2 0 + b_3 X_{1i} 0 = b_0 + b_1 X_{1i}$

When group = 1:
$$\hat{Y}_i = b_0 + b_1 X_{1i} + b_2 X_{2i} + b_3 X_{1i} X_{2i} = b_0 + b_1 X_{1i} + b_2 1 + b_3 X_{1i} 1 = (b_0 + b_2) + (b_1 + b_3) X_{1i}$$

A note on extra SS.

SAS recognizes 4 types of sum of squares in various procedures (especially PROC GLM). However, only two types of SS apply to regression. These are called TYPE I SS (or sequential SS) and TYPE II SS (or partial SS). For regression TYPE III and TYPE IV are the same as TYPE II (partial SS).

For the SAS model: MODEL Y = X1 X2 X3 X4; SAS would fit the following TYPE I and TYPE II sums of squares.

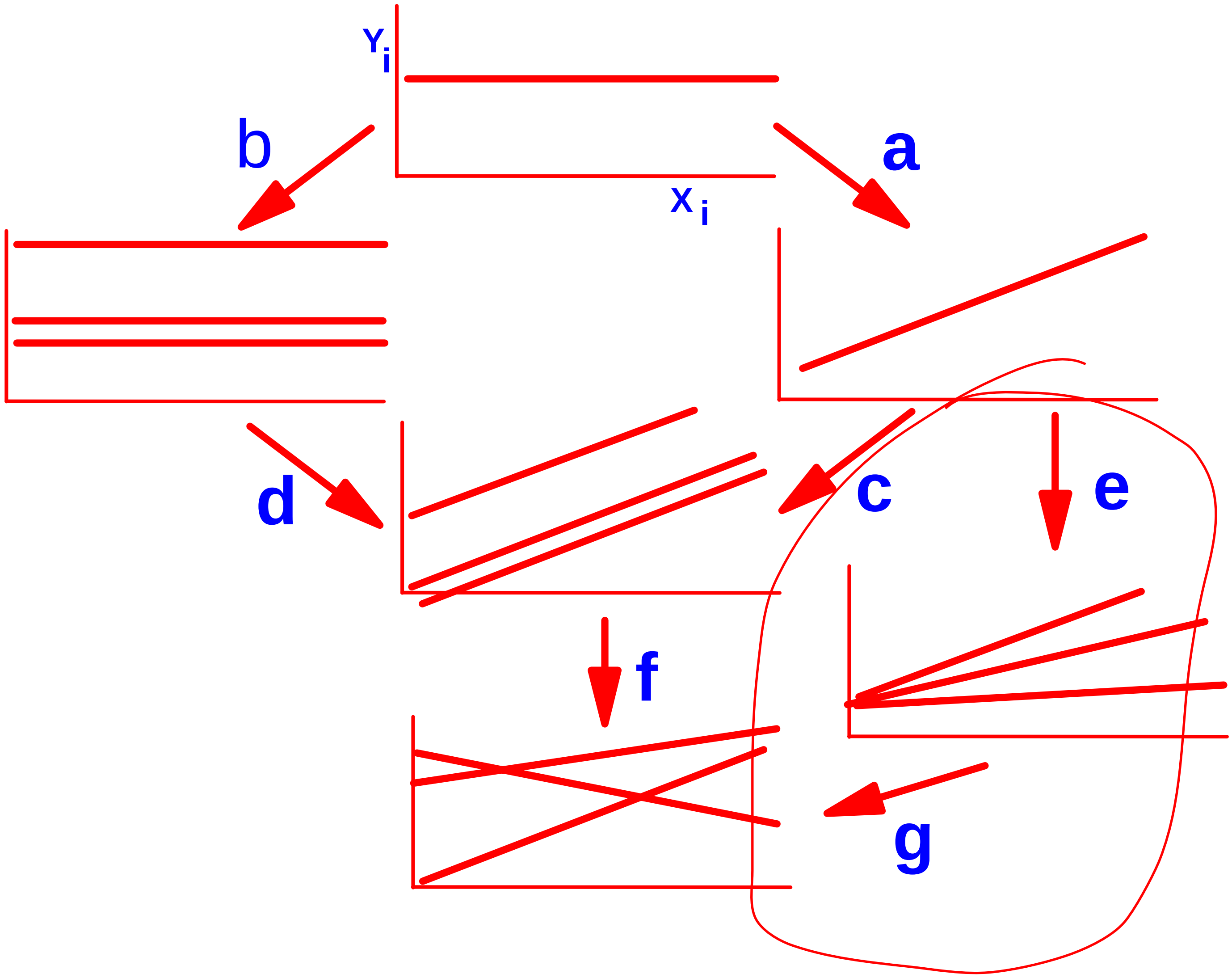
Variable	Type I SS	Type II, III or IV SS
X1	SSX1	SSX1|X2, X3, X4
X2	SSX2|X1	SSX2|X1, X3, X4
X3	SSX3|X1, X2	SSX3|X1, X2, X4
X4	SSX4|X1, X2, X3	SSX4|X1, X2, X3

Indicator variables – Non quantitative variables, called CLASS variables, GROUP variables or indicator variables are ANOVA type variables. These distinguish between groups such as freshman, sophomore, junior and senior or Male and Female. They require, as a group, one less degree of freedom than there are groups, as we saw in ANOVA (i.e. t groups require t – 1 d.f.)

These variables are coded in the analysis as 0 and 1, similar to the contrasts we saw in ANOVA. Also, as with ANOVA, the indicator variable will fit the difference between means for the various groups. When included in regression the indicator variable will fit differences in levels or intercepts.

Indicator variables are usually treated as a group, so SAS will report the SS for the group of variables. If, for example, we had the CLASS variable "YEAR" with levels [freshman, sophomore, junior and senior], SAS would calculate a single sum of squares for the group with 3 d.f.

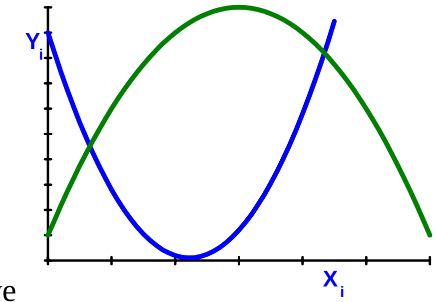
Analysis of Covariance – a combination of quantitative and indicator variables



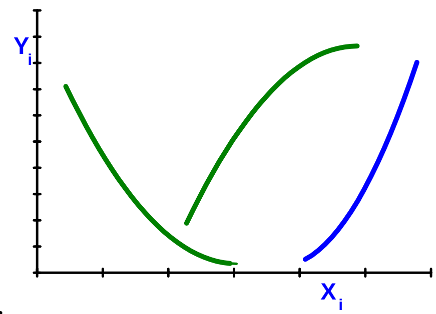
```
61         Title2 'Analysis of Covariance';
62         options ps=512 ls=111;
63         PROC GLM DATA=Meadowfoam;
64            MODEL Flowers = Intensity time0 intensity*time0;
65         RUN;
66         quit;
NOTE: The PROCEDURE GLM printed pages 10-11.
NOTE: PROCEDURE GLM used (Total process time):
      real time           0.09 seconds
      cpu time            0.04 seconds
67         ODS HTML close;

Chapter 9 : The effect of light on Meadowfoam flowering
Analysis of Covariance

The GLM Procedure

Number of Observations Read          24
Number of Observations Used          24

Dependent Variable: FLOWERS
                                        Sum of
Source                      DF         Squares     Mean Square    F Value    Pr > F
Model                        3     3467.276422     1155.758807      26.55    <.0001
Error                       20      870.659845       43.532992
Corrected Total             23     4337.936267

R-Square      Coeff Var      Root MSE    FLOWERS Mean
0.799292       11.75320      6.597954        56.13750

Source                      DF       Type I SS     Mean Square    F Value    Pr > F
INTENSity                    1     2579.750045     2579.750045      59.26    <.0001
Time0                        1      886.950342      886.950342      20.37    0.0002
INTENSity*Time0              1        0.576035        0.576035       0.01    0.9096

Source                      DF     Type III SS     Mean Square    F Value    Pr > F
INTENSity                    1     1328.712043     1328.712043      30.52    <.0001
Time0                        1      153.216013      153.216013       3.52    0.0753
INTENSity*Time0              1        0.576035        0.576035       0.01    0.9096

                                        Standard
Parameter                Estimate          Error    t Value    Pr > |t|
Intercept             71.62333349     4.34330481      16.49      <.0001
INTENSity             -0.04107619     0.00743505      -5.52      <.0001
Time0                 11.52333336     6.14236056       1.88      0.0753
INTENSity*Time0        0.00120952     0.01051475       0.12      0.9096
```

Polynomials – models employing successive power terms (all terms must be included up to the highest power used in the model.) These should be fitted with TYPE I SS.

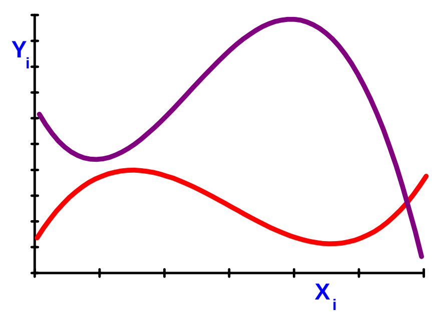
$$\mu_{\{Y|X,X^2X^3\ldots X^k\}} = \beta_0 + \beta_1 X_i + \beta_2 X_i^2 + \beta_3 X_i^3 + \ldots + \beta_k X_i^k$$



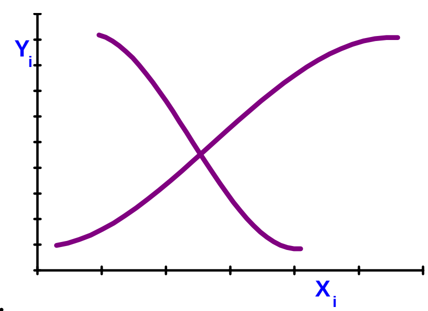
Polynomials: With X and X^2 it is called a Quadratic curve



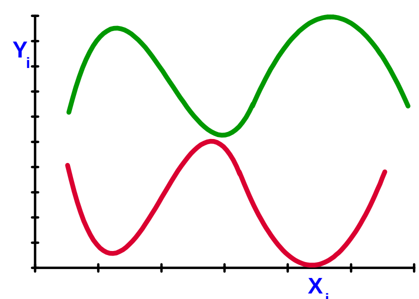
It is not necessary to fit the full sweep of the curve.



Cubic models have X, X^2 and X^3.



Again, it is not necessary to fit the full sweep of the curve.



Quartic model (X, X^2, X^3 and X^4):