Analysis of Variance [Chapter 5, part 2]
A second case of analysis of variance, the Dr. Spock conspiracy trail.
This case is an observational study, so the data does not come from a planned experiment, conducted under controlled conditions.

The claim by the defense is that the number of women (who might favor Dr. Spock) was underrepresented. In fact, his jury had no women. There were 7 U. S. District Court judges in the Boston area, including Dr. Spock’s judge. The null hypothesis is that the mean number of women for the 7 judges is equal ( $H_{0}: \mu_{1}=\mu_{2}=\mu_{3}=\mu_{4}=\mu_{5}=\mu_{6}=\mu_{7}$ ) versus the alternative (some $\mu_{\mathrm{i}}$ is different).

## Display 5.5 Percentages of women on venires of the seven Boston judges



Although an observational study, the analysis is the same. Using PROC MIXED and PROC GLM to do our Analysis of Variance we get the following.

Chapter 5 : Spock Conspiracy Trial
Analysis of variance with PROC GLM
The GLM Procedure

| Class Level |  |  |  |  | Information |
| :--- | ---: | ---: | ---: | :---: | :---: |
| Class | Levels | Values |  |  |  |
| Judge | 7 | A B C D E F SPOCK |  |  |  |
|  |  |  |  |  |  |
| Number of Observations Read | 46 |  |  |  |  |
| Number of Observations Used | 46 |  |  |  |  |

## Dependent Variable: Percent



What conclusion can be made from these results? Clearly the $F$ value of 6.72 would be unusual under the null hypothesis, and would occur by random chance with a probability of less than one in 10,000. The null hypothesis ( $\mathrm{H}_{0}: \mu_{1}=\mu_{2}=\mu_{3}=\mu_{4}=\mu_{5}=\mu_{6}=\mu_{7}$ ) would be rejected for the alternative (some $\mu_{\mathrm{i}}$ is different).
For these relatively simple problems both PROC MIXED and PROC GLM should give the same results.
Chapter 5 : Spock Conspiracy Trial
Analysis of variance with PROC MIXED
The Mixed Procedure
Model Information
Data Set
WORK. JURY
Dependent Variable
Covariance Structure
Percent

Estimation Method
Residual Variance Method
Fixed Effects SE Method
Diagonal
REML
Profile
Degrees of Freedom Method Residual
Class Level Information
Class Levels Values
Judge $7 \quad$ A B C D E F SPOCK
Dimensions
Covariance Parameters 1
Columns in X 8
Columns in Z 0
Subjects 1
Max Obs Per Subject 46
Number of Observations
Number of Observations Read 46
Number of Observations Used 46
Number of Observations Not Used 0

```
Covariance Parameter Estimates
Cov Parm Estimate
Residual 47.8063
```


## Fit Statistics

| -2 Res Log Likelihood | 274.0 |
| :--- | :--- |
| AIC (smaller is better) | 276.0 |
| AICC (smaller is better) | 276.1 |
| BIC (smaller is better) | 277.6 |


| Type 3 Tests of Fixed Effects |  |  |  |
| :---: | ---: | ---: | ---: |
| Num | Den |  |  |
| DF | DF | F Value | Pr $>$ F |
| 6 | 39 | 6.72 | $<.0001$ |

Note that these results match those of the GLM. For some more complicated models and some other types of problems, this will not be true.
Checking the assumptions: Here, as with the first example we will examine the residuals for normality and homogeneity of variance.

First, the PROC UNIVARIATE with the BY JUDGE statement provides a plot similar to that provided by the book.


From this plot we can see box plots of the individual group members (judges). Note that there are few potential outliers and no consistent indication of skewness (mean $<$ or $>$ the median). Some judges seem to have relative large variability while other are smaller. This may indicate nonhomogenous variance.

The dataset below is the output from the PROC GLM OUTPUT statement. Many SAS procs have facilities for outputting results from the procedure. We had previously seen the "OUTP=somename" option on the PROC MIXED MODEL statement. The style "OUTPUT" below is more common is SAS.

```
44 proc glm data=Jury;
45 class judge;
46 model percent = judge;
47 output out=next1 r=e p=yhat;
4 8
run;
```

With this statement it is possible to specify names for key variables (keyvariable=somename). The key variable names include the following: P or PREDICTED, R or RESIDUAL, RSTUDENT, STUDENT, L95, L95M, U95 and U95M.

| Chapter 5 : Spock Conspiracy Trial |  |  |  |  | 21 | 24.3000 | D | -2.7000 | 27.0000 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Analysis among judges |  |  |  |  | 22 | 29.7000 | D | 2.7000 | 27.0000 |
| Analysis of variance with PROC GLM |  |  |  |  | 23 | 17.7000 | E | -9.2667 | 26.9667 |
| Listing of PROC MIXED residuals |  |  |  |  | 24 | 19.7000 | E | -7.2667 | 26.9667 |
|  |  |  |  |  | 25 | 21.5000 | E | -5.4667 | 26.9667 |
| Obs | Percent | Judge | e | yhat | 26 | 27.9000 | E | 0.9333 | 26.9667 |
| 1 | 16.8000 | A | -17.3200 | 34.1200 | 27 | 34.8000 | E | 7.8333 | 26.9667 |
| 2 | 30.8000 | A | -3.3200 | 34.1200 | 28 | 40.2000 | E | 13.2333 | 26.9667 |
| 3 | 33.6000 | A | -0.5200 | 34.1200 | 29 | 16.5000 | F | -10.3000 | 26.8000 |
| 4 | 40.5000 | A | 6.3800 | 34.1200 | 30 | 20.7000 | F | -6.1000 | 26.8000 |
| 5 | 48.9000 | A | 14.7800 | 34.1200 | 31 | 23.5000 | F | -3.3000 | 26.8000 |
| 6 | 27.0000 | B | -6.6167 | 33.6167 | 32 | 26.4000 | F | -0.4000 | 26.8000 |
| 7 | 28.9000 | B | -4.7167 | 33.6167 | 33 | 26.7000 | F | -0.1000 | 26.8000 |
| 8 | 32.0000 | B | -1.6167 | 33.6167 | 34 | 29.5000 | F | 2.7000 | 26.8000 |
| 9 | 32.7000 | B | -0.9167 | 33.6167 | 35 | 29.8000 | F | 3.0000 | 26.8000 |
| 10 | 35.5000 | B | 1.8833 | 33.6167 | 36 | 31.9000 | F | 5.1000 | 26.8000 |
| 11 | 45.6000 | B | 11.9833 | 33.6167 | 37 | 36.2000 | F | 9.4000 | 26.8000 |
| 12 | 21.0000 | C | -8.1000 | 29.1000 | 38 | 6.4000 | SPOCK'S | -8.2222 | 14.6222 |
| 13 | 23.4000 | C | -5.7000 | 29.1000 | 39 | 8.7000 | SPOCK'S | -5.9222 | 14.6222 |
| 14 | 27.5000 | C | -1.6000 | 29.1000 | 40 | 13.3000 | SPOCK'S | -1.3222 | 14.6222 |
| 15 | 27.5000 | C | -1.6000 | 29.1000 | 41 | 13.6000 | SPOCK'S | -1.0222 | 14.6222 |
| 16 | 30.5000 | C | 1.4000 | 29.1000 | 42 | 15.0000 | SPOCK'S | 0.3778 | 14.6222 |
| 17 | 31.9000 | C | 2.8000 | 29.1000 | 43 | 15.2000 | SPOCK'S | 0.5778 | 14.6222 |
| 18 | 32.5000 | C | 3.4000 | 29.1000 | 44 | 17.7000 | SPOCK'S | 3.0778 | 14.6222 |
| 19 | 33.8000 | C | 4.7000 | 29.1000 | 45 | 18.6000 | SPOCK'S | 3.9778 | 14.6222 |
| 20 | 33.8000 | C | 4.7000 | 29.1000 | 46 | 23.1000 | SPOCK'S | 8.4778 | 14.6222 |

Refer to the SAS output for evaluation of the assumptions. In particular, note the following concerning assumptions.

1) Is the assumption of normality met?
2) Are there any outliers?
3) Is there a suggestion of non-homogeneous variance in the residuals?
