Statistical Techniques I

EXST7005

Factorial Treatments & Interactions

The Factorial Treatment Arrangement

- Also known as "two-way" ANOVA
  - This analysis has two (or more) Treatments, for example treatment A with two levels (a1 and a2) and treatment B with two levels (b1 and b2).
  - Each level of one treatment occurs with each level of the other treatment (cross-classified)
  - e.g. a1b1, a1b2, a2b1, a2b2
  - Each treatment may be fixed or random (independently)

The Factorial Treatment Arrangement *(continued)*

- Combinations of treatments are assigned at random to experimental units, so the design is still a CRD (there are 4 combinations in the example given (a1b1, a1b2, a2b1, a2b2)
- The treatment arrangement is called a "factorial", and the dimensions are usually given as 2 by 2 (above), 2 by 3, 3 by 3, etc.
-

Schematic of a 3 by 3 factorial

Treatments	A1	A2	A3
B1	a1b1	a2b1	a3b1
B2	a1b2	a2b2	a3b2
B3	a1b3	a2b3	a3b3

Marginal sums or means

Treatments	A1	A2	A3	Sum
B1	a1b1	a2b1	a3b1	b1 sum
B2	a1b2	a2b2	a3b2	b2 sum
B3	a1b3	a2b3	a3b3	b3 sum
Sum	a1 sum	a2 sum	a3 sum	

Interactions

- The principle treatments (A and B in the previous examples) are called main effects.
- The main effect for treatment A will be calculated from the marginal of that treatment, averaged across the B treatment.
- Likewise, the main effect of treatment B will be calculated from the marginals for treatment B.

Interactions *(continued)*

- Calculations for the main effects (Uncorrected treatment SS) are the same as for the CRD.
- There is however one new issue. It is possible for the same main effects to arise from various different cell patterns.
-

Interactions *(continued)*

Treatment	a1	a2	a3	a4	Means
b1	2	5	10	3	5
b2	4	7	12	5	7
Means	3	6	11	4	

Treatment	a1	a2	a3	a4	Means
b1	2	3	12	3	5
b2	4	9	10	5	7
Means	3	6	11	4	

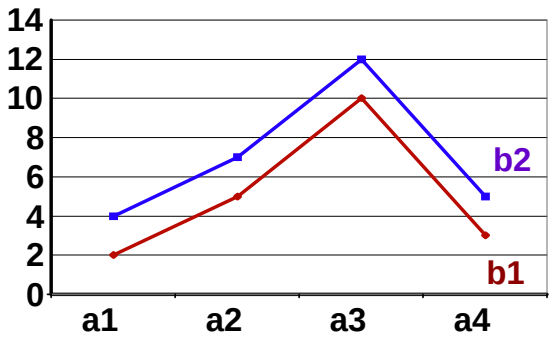
Interaction effects

Treatment	a1	a2	a3	a4	Effect
b1	2	5	10	3	-1
b2	4	7	12	5	+1
Effect	-3	0	+5	-2	(6)

Treatment	a1	a2	a3	a4	Effect
b1	2	3	12	3	-1
b2	4	9	10	5	+1
Effect	-3	0	+5	-2	(6)

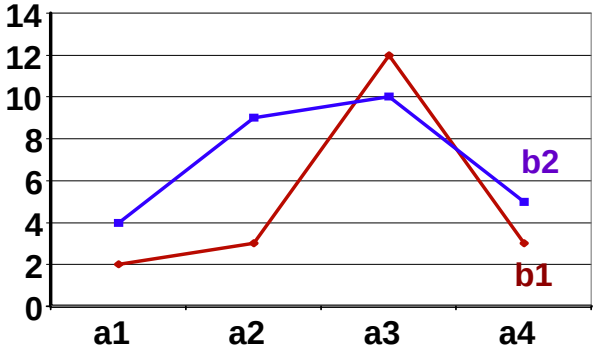
Interactions *(continued)*

- Plotting the means for the first case.



Interactions *(continued)*

- Plotting the means for the second case.



Interactions *(continued)*

- This lack of consistency in the cells is caused when the marginals are not strictly additive.
- When additivity exist if some treatment marginal mean (#1) is larger by 2 units than some other mean (#2), each cell will in treatment #1 be 2 units higher than the corresponding mean of the treatment #2.
- This would represent additivity, or no interaction between the treatments.

Interactions *(continued)*

- If, however, the increases and decreases are not consistent, with the marginal means, then there is an interaction, or a lack of additivity.
- The marginal means (or sums) are used to calculate the main effects of the treatments. The cell to cell variation is used to measure the interaction (after adjusting for the main effects).

Interactions *(continued)*

- If we plot the treatment means, as done previously, and the lines do not appear parallel, then there is some interaction.
- However, the lines are never perfectly parallel. Is the departure from additivity significant or not?
- To determine this, we test the interaction. This is normally done in ANOVA for all factorial designs.

Interactions *(continued)*

- Interpreting interactions
  - Sometimes the main effects are very important relative to the interactions and may tell you most of what you need to know.
  - Sometimes interactions are more important
  - However, significant interactions indicate that the main effects are somehow inconsistent. You should determine how this inconsistency affects your eventual conclusions.
  - Significant interactions should not be ignored.

Factorial contrasts

- Factorial experiments, or two-way ANOVAs, are usually done in SAS by entering two class variables and their interaction in the model.
- e.g. PROC GLM; CLASSES A B;
- MODEL Y = A B A*B; RUN;

Factorial contrasts *(continued)*

- However, it is also possible to do factorials as contrasts, setting up the treatments as a one-way ANOVA.
- For a simple 2 by 2 factorial, with treatments A and B, we have a total of 4 cells and 3 degrees of freedom.
- The 4 combinations are
  - a1b1, a2b1, a1b2 and a2b2.

Factorial contrasts *(continued)*

- We can test the A main effect with a contrast, likewise the B main effect. To test he interaction, calculate the cross-product of the A and B contrasts.

Tmt	a1b1	a1b2	a2b1	a2b2	Sum
A	-1	-1	1	1	0
B	-1	1	-1	1	0
A*B Inter	1	-1	-1	1	0

Factorial contrasts *(continued)*

- For larger designs the pattern is similar, for example the 2 by 2 by 2 below.
- Treatment A has two levels (a and A), B has two levels (b and B) and C has levels (c and C).
- All contrasts consist of plus ones or minus ones, so only the + or - is shown.

Factorial contrasts *(continued)*

Tmt	abc	Abc	aBc	ABc	abC	AbC	aBC	ABC	Sum
A main	-	+	-	+	-	+	-	+	0
B main	-	-	+	+	-	-	+	+	0
A * B	+	-	-	+	+	-	-	+	0
C main	-	-	-	-	+	+	+	+	0
A * C	+	-	+	-	-	+	-	+	0
B * C	+	+	-	-	-	-	+	+	0
A * B * C	-	+	+	-	+	-	-	+	0

Factorial contrasts *(continued)*

- A larger factorial. with more than 2 levels in some treatment, would have more than 2 d.f. in some treatment.
- This would require a 2 or 3 or more d.f. contrast. These can be done in SAS but we will not discuss these this semester.

Summary

- Factorials, or two-way ANOVA, was covered.
  - A factorial is a way of entering two or more treatments into an analysis.
  - The description of a factorial usually includes a measure of size, a 2 by 2, 3 by 4, 6 by 3 by 4, 2 by 2 by 2, etc.
  -

Summary *(continued)*

- Interactions were discussed.
  - Interactions test additivity of the main effects
  - interactions are a measure of inconsistency in the behavior or the cells relative to the main effects.
  - Interactions are tested along with the main effects
  - Interactions should not be ignored if significant.

Summary *(continued)*

- Factorial analyses
  - can be done as two-way ANOVAs in SAS,
  - or they can be done as contrasts.
-